The strain dependence of the critical properties of Nb$_3$Sn conductors

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The critical current ($I_c$) of six different Nb$_3$Sn multifilamentary wires is investigated as a function of temperature, magnetic field, and strain. A relation for a critical temperature ($T_c$) that depends on the deviatoric strain is proposed and applied to interpret the results. First, a short review is given on the flux-pinning relations that are used to introduce a strain dependent $T_c$ in a relation for the $I_c$ as a function of field and temperature. The conductor samples are investigated in two different deformation states, namely, in a spiraled shape on a Ti sample holder and a straight section soldered onto a brass substrate. The brass substrate is used to apply a compressive or tensile axial strain to the conductor. The $I_c$ in the different samples prepared from a single conductor type can be described very well with a single set of critical properties and strain parameters. In particular, in the strain regime where the matrix deformation is limited and the superconductor is axially compressed, the proposed strain relation is very accurate. The small variation in the strain parameter between the six conductors investigated suggests that this strain parameter is an intrinsic property of Nb$_3$Sn.

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I. INTRODUCTION

The relation between a mechanical deformation and the superconducting properties of Nb$_3$Sn and other A15 materials is an interesting topic that has been studied for many years. The dependence of the critical temperature ($T_c$) and the upper critical field ($B_{c2}$) on hydrostatic pressure ($P$) is considered in descriptions for the superconducting state. An important property of A15 superconductors is that a first-order description for the hydrostatic pressure dependence of $T_c$ or $B_{c2}$ is not adequate to describe the changes in the critical current density ($J_c$) of an axially deformed superconductor. For the $J_c$ of an axially elongated A15 conductor there is a scaling formula developed by Ekin. Analyzing the available experimental results on deformed superconductors, Welch concluded that a nonhydrostatic component of the strain tensor is the most important factor that determines the critical properties in deformed A15 superconductors.

A. Deformation in composite conductors

The intrinsic strain inside the superconducting filament(s) or layer(s) of a technical superconductor is determined by the mechanical interaction with the matrix material(s) and the experimental setup. In the case of an A15 conductor, it is crucial to consider the thermally induced strain over the entire temperature trajectory from the heat-treatment temperature down to the operating temperature. In a typical Nb$_3$Sn wire conductor an axially compression of 0.05%–0.4% is induced in the Nb$_3$Sn filaments by the matrix when the conductor is cooled down to $T=4.2$ K in a force-free state. When mounted on a metallic substrate, or embedded in a cable inside a stainless-steel conduit, this thermally induced compression in the Nb$_3$Sn can be even twice as high.

The large difference in thermal contraction between the materials in a composite conductor makes it practically impossible to obtain a true strain-free state inside the filaments of such a superconductor. When, for instance, the axial strain component is reduced to zero, then the other (off-axis) components of the strain tensor will, in general, not be zero inside the superconducting filament. A second consequence of the large thermal contraction differences between the superconductor and the connected matrix is that the strain is always relatively high inside a composite conductor, compared to the limits for elastic deformation. Therefore, it is likely that nonelastic deformations (yielding or cracks) will occur either in the superconductor or in the matrix.

B. Nonhydrostatic deformations in Nb$_3$Sn conductors

A number of deformation experiments have been performed on Nb$_3$Sn tapes to investigate the influence of the nonhydrostatic strain on the critical properties in more detail. In this experiment, the tape geometry is selected because it enables a more precise determination of all the strain components involved. The experimental results obtained on tape conductors confirmed the dominant role of the nonhydrostatic deformation on the critical properties of Nb$_3$Sn as originally proposed by Welch. A rise in $J_c$ and $B_{c2}$ is induced by applying a transverse pressure to a Nb$_3$Sn tape that has a large thermal precompression along the in-plane directions in the Nb$_3$Sn layer. This pressure-induced rise in $J_c$ and $B_{c2}$ can be explained precisely with nonhydrostatic strain components in the Nb$_3$Sn layer. The results of a (semi) two-component deformation experiment on bendable substrates are also described very well with a formulation based on nonhydrostatic strain components in the Nb$_3$Sn layer.

In this article we present results on axially deformed Nb$_3$Sn superconductors. The results are evaluated with a de-
scription that correlates the critical properties to the devia-
toric strain tensor. First, a short review is presented in order
to connect this deviatoric strain dependence with the maximum
scaling formulas for the maximum pinning force and the
dependence of \( J_c \) on the magnetic field and temperature. Af-
after a description of the experimental conditions, the critical
current of six different types of Nb3Sn conductors is evaluated
as a function of the temperature, magnetic field, and axial strain.

II. THEORY

The description for the critical current density of type II
superconductors is based on the critical state concept, in
which the critical current density is related to the maximum
pinning force \( F_p \) that depends on the magnetic field \( (B) \) and
temperature \( (T) \). In that case, \( J_c \) is determined by \( ^6 \)

\[
J_c(B,T) \times B = -F_p(B,T) = -C \kappa(T)^{-\gamma}B_{c2}(T)^{\nu}f(B/B_{c2}(T)),
\]

with \( 2 \leq \nu \leq 3 \), but comes closest to a value of 2. The func-
tion \( \kappa(T)^{-\gamma} \) introduces a temperature-dependent Ginzburg–
Landau parameter \( \kappa \), with \( 1 < \gamma < 3 \) while the field depen-
dence in this relation is determined by the function \( f \).

A. Flux pinning and strain dependence

Ekin \(^1\) stated that the influence of mechanical deforma-
tions on the pinning force should be written in the same
explicit way as the temperature dependence:

\[
F_p(B,e) = C B_{c2}(e)^{\nu}f(B/B_{c2}(e)),
\]

with \( n = 1 \pm 0.3 \) for a measurement at a reference tem-
perature (e.g., 4.2 K). The strain dependence of \( B_{c2} \) is described
with the function \( S(e) = B_{c2}(4.2,e)/B_{c2m}(4.2) \), as an empiri-
cal fit of the \( B_{c2}(e) \) data at 4.2 K scaled to the maximum
\( (B_{c2m}) \) in the strain dependency curve. The field dependence
\( f(B/B_{c2}) \) included in this relation is the pinning relation
originally proposed by Kramer \(^9\):

\[
f(B/B_{c2}(T,e)) = f(b) = b^p(1-b)^q,
\]

with \( p \approx 0.5 \) and \( q = 2 \) as typical values for Nb3Sn. In order
to combine Eqs. (1) and (2), it is necessary to include the strain
dependence of \( T_c \).

The strain dependence of \( T_c \) is related with the strain
dependence of \( B_{c2} \) at 4.2 K according to \(^1,2\)

\[
\frac{T_c(e)}{T_{cm}} = \left( \frac{B_{c2}(4.2,e)}{B_{c2}(4.2)} \right)^{1/w},
\]

where \( w \approx 3 \) for A15 materials. With the condition that \( S(e) \) is independent of
temperature, this leads to the following relations for \( T_c \) and \( B_{c2} \):

\[
T_c(e) = T_{cm}S(e)^{1/w}, \quad B_{c2}(T,e) = B_{c2m}(0) \cdot S(e) \beta(T,e).
\]

The factor \( \beta(T,e) \) in these relations defines the temperature
dependence of \( B_{c2} \) as

\[
\beta(T,e) = [1 - (T/T_c(e))] K(T,e),
\]

which includes the temperature dependence of the
Ginzburg–Landau parameter in a factor \( K(T,e) \) that can be expressed in the approximation proposed by Summers \(^10\):

\[
K(T,e) = 1 - 0.31 \left[ T/T_c(e) \right]^2 \left[ 1 - 1.77 \ln \left[ T/T_c(e) \right] \right].
\]

Finally, the \( J_c(B,T,e) \) relation is then expressed as

\[
J_c(B,T,e) = \frac{C \beta(T,e)^{\nu}S(e)^{\nu}f(B/B_{c2}(T,e))}{BK(T,e)^{2}(4.2)}.
\]

This relation describes the critical current density with three
material parameters: \( T_{cm} \) is the critical temperature at the maximum in the strain curve at zero applied field, \( B_{c2m}(0) \) is the upper-critical field at 0 K at the maximum of the strain curve, and \( C \) is a scaling constant for the maximum pinning
force that is proportional to the critical current density. The parameter \( e \) in the strain dependence \( S(e) \) is an effective
value representing the intrinsic state of strain that is present
in the superconductor at the operating temperature. Possible
formulations for the strain function \( S(e) \) are considered in
the next section.

B. The deviatoric strain description

Axial strain experiments on A15 conductors are de-
scribed well by a power-law dependence for the axial strain
dependence function \( S(e_{ax}) \) as given by Ekin \(^1\):

\[
S(e_{ax}) = 1 - a |e_{ax} - e_m|^\alpha,
\]

where \( e_m \) is equal to the applied axial strain \( e_{ax} \) at which the maximum in \( J_c \) occurs. This power-law relation \( S(e_{ax}) \) describes the experimental results on axially elongated wires
very well, if two different values for the strain-scaling con-
stant \( a \) are used. Typical values for Nb3Sn are \( a = 900 \) for
\( e_{ax} < e_m \) and \( a = 1250 \) for \( e_{ax} > e_m \), with a constant value for
the exponent \( (n = 1.7) \).

For a more complete three-dimensional description, the
entire strain tensor has to be considered in the strain function
\( S(e) \). The nonhydrostatic strain can be represented by the
second strain invariant of the deviatoric strain tensor. In a
rectangular coordinate system with the principal strain axis
coinciding with the coordinate axes \( (x,y,z) \), this strain com-
ponent can be represented as

\[
e_m = \frac{1}{3} \sqrt{\frac{3}{2} (e_{xx} - e_m)^2 + (e_{yy} - e_m)^2 + (e_{zz} - e_m)^2},
\]

where \( e_{xx}, e_{yy}, \) and \( e_{zz} \) represent the (plane) strain in the prin-
ciple directions inside the material. By considering only this
particular strain component \( e_{dev} \) (referred to as the “deviatoric
strain”) a three-dimensional strain dependence is proposed:

\[
S(e_{dev}) = \frac{1 - C_d (e^{dev}_{ax})^2 + (e_{dev})^2}{1 - C_d (e_{dev})^2} \approx 1 - C_d e_{dev},
\]

where \( e_{dev}^0 \) is a constant that describes the exact shape of
\( S(e_{dev}) \) for small strains \( (e_{dev} < e_{dev}^0) \). This factor appears to
be small compared to \( e_{dev} \) in the experiments on Nb3Sn
tapes. This implies that the approximate formulation
\( (1 - C_d e_{dev}) \) adequately describes the experimental results on
with deformation experiments. The voltage is measured via taps over a length of tin during the heat treatment, both ends of the samples are provided by the manufacturer. In order to prevent leakage of batch under vacuum conditions according to the schedule samples of each type of strand are heat treated in a single on a Ti–6Al–4V sample holder that is depicted in Fig. 1. All function is equivalent to the deviatoric strain dependence of prestrain in the conductor. This formulation of the strain constant $e_a$ is minimized, at

$e = \frac{1 - C_a \sqrt{(e_a + \delta)^2 + (e_{0,a})^2}}{1 - C_a e_{0,a}}$.

In an axially deformed sample, the strain constant $C_a$ is proportional to $C_d$, but it is also determined by the sample-specific parameters $\nu_s$ and $\nu_y$. The maximum in $S$ occurs when the deviatoric strain is minimized, at $e_a = -\delta$. The constant $e_{0,a}$ represents the strain components that are still present inside the superconductor when $e_{dev}$ is minimized, as well as the factor $e_{0,d}$ mentioned previously. The exact value for $e_{0,a}$ is, therefore, determined by $\nu_s$, $\nu_y$, and the thermal prestrain in the conductor. This formulation of the strain function is equivalent to the deviatoric strain dependence of $B_z$ in Nb$_3$Sn wire conductors at a constant temperature as proposed earlier with a slightly differently defined scaling constant $C_a$.

III. EXPERIMENTAL SETUP

The results obtained on two types of experimental setups are compared. The critical current of the samples is determined on a "standard" sample holder that is used in many different experiments. These $I_c(B,T)$ values are compared with deformation experiments $I_c(e_a)$ at selected values for the applied magnetic field and temperature.

A. Critical current measurements

For the $I_c(B,T)$ measurements the strand is heat treated on a Ti–6Al–4V sample holder that is depicted in Fig. 1. All samples of each type of strand are heat treated in a single batch under vacuum conditions according to the schedule provided by the manufacturer. In order to prevent leakage of tin during the heat treatment, both ends of the samples are extended by a few centimeters and squeezed. After the heat treatment, the sample is fixed on the sample holder with epoxy. The voltage is measured via taps over a length of 500 mm and a criterion of $10^{-5}$ V/m is used to determine the $I_c$.

For the temperature-dependent measurements the sample holder is enclosed in a gas environment by covering the sample with an insulating cup. The sample temperature is controlled by a set of heaters and thermometers connected to the holder. During the measurement of the voltage–current transition the temperature at both sides of the sample is stabilized within 5 mK. Summarizing all the possible errors, the maximum uncertainty in the temperature error is $\pm 30$ mK at $B=0$ and $\pm 40$ mK at high magnetic fields. One set of $I_c$ measurements is performed in a magnetic field ranging from 7 to 13 T, with the samples in liquid He at atmospheric pressure (4.2 K). A second set of samples is investigated in the temperature range from 5 K up to 8 K, in a constant magnetic field of 13 T.

B. Deformation experiments

The setup to characterize the $I_c(B,T,e_a)$ dependency is shown in Fig. 2. After the heat treatment on a stainless-steel sample holder, the sample is transferred and soldered tightly to the brass sample holder with Sn–Ag for the measurements. Strain is applied by bending the U-shaped substrate and the sample at 4.2 K. This technique was also applied in previous experiments on similar conductors. Starting from the initial strain state at 4.2 K, the substrate is bent by means of a force that acts on the legs of the U-shaped substrate.

The strain in the sample is determined with two strain gauges that are connected to the central section of the substrate. The exact procedure to determine the applied strain as
TABLE I. Material parameters for Nb₃Sn.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>1</td>
</tr>
<tr>
<td>p</td>
<td>0.5</td>
</tr>
<tr>
<td>q</td>
<td>2</td>
</tr>
<tr>
<td>v</td>
<td>2</td>
</tr>
<tr>
<td>γ</td>
<td>1</td>
</tr>
<tr>
<td>w</td>
<td>3</td>
</tr>
</tbody>
</table>

TABLE II. The parameters as determined for all six conductors.

<table>
<thead>
<tr>
<th>Cond</th>
<th>Process</th>
<th>Cu/ non-Cu</th>
<th>Diff. Barr.</th>
<th>Tern. Addit.</th>
<th>RRR</th>
<th>( \delta_{\text{Ti}} ) (%)</th>
<th>( \delta_{\text{brass}} ) (%)</th>
<th>( \epsilon_{\text{sc}} ) (%)</th>
<th>( C_a )</th>
<th>( B_{\text{z,m}}(0) ) (T)</th>
<th>( T_{\text{cm}} ) (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Bronze</td>
<td>1.49</td>
<td>Ta</td>
<td>...</td>
<td>147</td>
<td>-0.22</td>
<td>-0.51</td>
<td>0.12</td>
<td>38.0</td>
<td>33.3</td>
<td>17.8</td>
</tr>
<tr>
<td>B</td>
<td>Int. tin</td>
<td>1.38</td>
<td>Nb/Ta</td>
<td>1%Ti</td>
<td>80</td>
<td>-0.21</td>
<td>-0.60</td>
<td>0.14</td>
<td>37.4</td>
<td>31.2</td>
<td>17.0</td>
</tr>
<tr>
<td>C</td>
<td>Int. tin</td>
<td>1.59</td>
<td>Ta</td>
<td>...</td>
<td>130</td>
<td>-0.10</td>
<td>-0.62</td>
<td>0.26</td>
<td>41.5</td>
<td>29.3</td>
<td>16.9</td>
</tr>
<tr>
<td>D</td>
<td>Bronze</td>
<td>1.49</td>
<td>Ta</td>
<td>7.5%Ta</td>
<td>150</td>
<td>-0.14</td>
<td>-0.59</td>
<td>0.08</td>
<td>37.4</td>
<td>32.2</td>
<td>17.4</td>
</tr>
<tr>
<td>E</td>
<td>Int. tin</td>
<td>1.84</td>
<td>Ta</td>
<td>...</td>
<td>144</td>
<td>-0.11</td>
<td>-0.63</td>
<td>0.19</td>
<td>40.8</td>
<td>27.8</td>
<td>18.1</td>
</tr>
<tr>
<td>F</td>
<td>Int. tin</td>
<td>1.61</td>
<td>Nb/Ta</td>
<td>1%Ti</td>
<td>213</td>
<td>-0.02</td>
<td>-0.43</td>
<td>0.17</td>
<td>39.1</td>
<td>28.9</td>
<td>16.7</td>
</tr>
</tbody>
</table>

IV. RESULTS

There is a large number of parameters involved in the \( J_c(B,T,\epsilon) \) relation considered here. A distinction can be made between the parameters that represent intrinsic properties of Nb₃Sn and the parameters that depend on the production route of the Nb₃Sn conductor and the sample preparation. The material properties, summarized in Table I, are considered to be constants for the Nb₃Sn in all the investigated conductors.

The critical properties \( B_{\text{z,m}}(0) \) and \( T_{\text{cm}} \) and the strain scaling parameters \( C_a \) and \( \epsilon_{\text{sc}} \) are expected to be constant for a certain type of Nb₃Sn production process. The thermal prestrain depends not only on the manufacturing process but also on the type of sample preparation and holder material. Therefore, two different values for \( \delta \) are used: \( \delta_{\text{Ti}} \) for the sample attached to the Ti barrel and \( \delta_{\text{brass}} \) for the samples soldered to the strain device. The prefactor \( C \) in the pinning relation (9) is proportional to the critical current. Any variation in \( C \) has exactly the same effect on \( I_c \) as a variation in the superconducting cross section \( (A_{sc}) \) inside the conductor.

Conductors from six different manufacturers (A–F) are investigated in this study. The emphasis is on the strain regime where the deformations in the matrix materials are low, from −0.4% to +0.4% applied axial strain. The properties of the strands are summarized in Table II. The \( I_c \) in a superconductor is proportional to the effective cross section \( (A_{sc}) \) and the pinning constant \( (C) \). This factor \( (A_{sc}C) \) is considered as sample dependent for the conductors that show a large variation in \( I_c \) among the samples.

A. The critical current in conductor A

The strain dependence of the \( I_c \) is measured on two samples from one conductor (A). The results of the measurements are summarized in Fig. 3. The measured \( I_c \) values coincide over the entire strain regime and all the investigated values of \( B \) and \( T \). In a large part of the regime, from −0.4% to +0.4% applied strain, the \( I_c \) changes nearly proportionally to the applied strain. The maximum in the \( I_c(\epsilon_{\text{sc}}) \) curve occurs at 0.51% applied strain for both samples. It is also visible that the measured \( I_c(\epsilon_{\text{sc}}) \) points are not perfectly symmetric around this maximum.

The \( I_c(B) \) and \( I_c(T) \) measurements on a third sample from the same conductor, are included in the Figs. 4 and 5.
In order to evaluate the description for $J_c(B,T,e)$ in Eq. (9) and the deviatoric strain relation presented in Eq. (13), a comparison is made with all the data measured on these three samples. The lines plotted with the measured data of this conductor in Figs. 3, 4, and 5 represent the set of parameters that is listed in Table II. It can be seen that the proposed relation describes the measurements very well over the entire measuring range and for all the three samples measured from this conductor.

The good correlation between all the $I_c$ values determined on conductor A supports the validity of the presented description for $J_c(B,T,e)$. The fact that the current amplitude correlates so well with a single value for the product $A_{sc}C$ is an important verification for the experimental procedures and in particular the sample preparation. An important limitation of the proposed semielastic description in the deviatoric strain dependence is visible around the maximum in $I_c(e_a)$. The asymmetry that is observed in the strain dependence of $I_c$ is not described accurately by this approximated mechanical model. The asymmetric behavior of $I_c$ around the maximum is considered in detail for all the investigated conductors in the next sections.

### B. Nonelastic deformations

An interesting behavior in the strain dependence around the $I_c$ maximum is observed in the experiments on conductor B, as presented in Fig. 6. Like in conductor A, the $I_c$ also changes nearly proportionally to the applied strain for $e_a < +0.4\%$. A single value for the product $A_{sc}C$ can be used to describe the $I_c$ dependence in the axially deformed samples as well as in the measurements on the Ti holder (see Figs. 4 and 5). Again, the corresponding parameters for Eq. (9) are given in Table II. The only deviations in the $I_c$ measurements occur at a large tensile strain for sample B1, where an increased reduction is observed.

In the strain regime around the maximum in $I_c$, the validity of the semielastic model becomes questionable. In particular, the assumption of a linear relation between the axial and off-axis strain components can be invalid. The samples are subjected to a large deformation trajectory: first the thermal contractions during cooldown and then the deformation from $-0.35\%$ to $0.70\%$ strain. This may result in nonelastic deformations like yielding and cracks. In sample B1 the deviations in $I_c$ occur around the point where the strain in the Nb$_3$Sn is minimized and the axial strain in the matrix materials becomes large ($>0.5\%$). This observation is a strong
measurements on the Ti holder is still good. Therefore, the $I_c$ can be described well with a single set of conductor parameters for the product $A_{ac}C$. In the case of conductors D, E, and F, Eq. (9) is accurate in the regime from $-0.4\%$ to $+0.4\%$, but the product $A_{ac}C$ is not exactly constant for the samples from one conductor. Nevertheless, these variations are relatively small. For conductors D, E, and F a variation of, respectively, $\pm 3\%$, $\pm 4\%$, and $\pm 10\%$ is observed between the samples investigated in $I_c(B)$ and $I_c(T)$ on the Ti barrel and the two samples investigated in the strain setup at four different combinations of $B$ and $T$.

D. The deviatoric strain scaling function

Several aspects can be mentioned in relation to the description of $I_c(B,T,\epsilon_a)$ that is based on the deviatoric strain. First of all, there is a large number of parameters listed in Table I, that can be considered as material constants for Nb$_3$Sn superconductors. Then, there are several properties that depend on the production process and the sample preparation. A few comments can be placed among the experimentally obtained parameters listed in Table II. The values found for the thermal prestrain ($\Delta_Ti$ and $\Delta_{gras}$) correlate well among the samples. The values deduced from the measurements on the two holders are very similar: $\Delta_Ti = -(0.16\pm0.06)\%$ and $\Delta_{gras} = -(0.57\pm0.06)\%$. Only sample F differs slightly different in this respect. In this conductor the large variation in $I_c$ between the samples complicates the analysis. The observed difference ($\Delta_Ti - \Delta_{gras} = 0.4\%$) is in good agreement with the prestrain difference that can be expected between these two preparation methods. The values determined for the critical properties ($B_{c2m}$ and $T_{cm}$) appear to be also realistic values.

The most intriguing result from these experiments is the small variation in the strain constant $C_a$ of only $\pm 6\%$ among the different conductors. The variation of the second strain parameter ($\epsilon_{0a}$) is comparable to the strain related parameters $\Delta_Ti$ and $\Delta_{gras}$. It should be noted that the physical background of the parameters $\epsilon_{0a}$ and $C_a$ is entirely different. The factor $\epsilon_{0a}$ determines the shape of $S(\epsilon)$ around the maximum, where the deviatoric strain is minimized. The complexity of the mechanical structure inside the sample, the variations in conductor layout, and the uncertainties in the deformation properties of the materials make it very difficult to predict $\epsilon_{0a}$ accurately. The strain constant $C_a$ determines the slope in $S(\epsilon)$ that occurs when the intrinsic axial strain in the superconductor ($\epsilon_a$) is large compared to $\epsilon_{0a}$. This linear part in $S(\epsilon)$ is correlated to the linear reduction in $B_{c2}$ or $T_{c}$ that was observed in axial compression experiments on Nb$_3$Sn, Nb$_3$Al, and V$_3$Si conductors. In this compressive strain regime, the power-law strain dependence [Eq. (10)] is not adequate to describe the $I_c$ in these conductors accurately.

The fact that the variation in parameter $C_a$ is small among these different conductors indicates that there is a correlation between the deviatoric strain and the critical properties for Nb$_3$Sn in general. The small variation observed in $C_a$ might be attributed to the differences in the mechanical behavior of the conductors. In particular, the ef-
fective overall Poisson’s ratio will depend on the conductor layout and the applied matrix materials. In this case, the constant $C_d$ that describes the deviatoric strain dependence of $S(e)$ can be regarded as an intrinsic material property of Nb$_3$Sn and not an arbitrary parameter that describes the strain dependence of a particular type of Nb$_3$Sn conductor. If $C_d$ is indeed a material constant for Nb$_3$Sn, then it enables a complete prediction of the strain dependence of the critical properties in an axially compressed conductor, only based on a determination of the critical properties at a single, well-defined, precompressed state.

V. CONCLUSIONS

(1) A relation for the critical current density of a deformed Nb$_3$Sn conductor is presented and experimentally verified at various temperatures and magnetic fields. The proposed relation is based on the deviatoric strain tensor. The connection between the three-dimensional deviatoric strain relation and an axially deformed wire conductor is obtained with a linear mechanical model.

(2) The proposed description is valid in the range where the matrix deformations are limited and the Nb$_3$Sn is compressed in the axial direction. The parameters that depend on the conductor type and the preparation method are: the critical properties, two constants describing the strain dependence ($C_a$ and $\epsilon_{0,a}$), and the initial thermal strain. All other parameters are considered constant for the Nb$_3$Sn in the investigated conductors.

(3) The proposed description does not describe the irregular behavior that is observed in conductors subjected to a large tensile axial strain. The variations in $I_c$ around its maximum and the differences observed between the samples of a single conductor type are attributed to nonelastic deformations. The $I_c$ in this strain regime is more accurately described by the asymmetric power-law dependence.

(4) The strain dependence of the conductors from various Nb$_3$Sn manufacturers is compared in detail at various temperatures and fields. The variation in the strain-scaling constant ($C_a$) is small. This suggests that the related deviatoric strain dependence ($C_d$) is an intrinsic property of Nb$_3$Sn.

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