CONTACT RESISTANCE AND NORMAL ZONE FORMATION
IN COATED YTTRIUM BARIUM COPPER OXIDE
SUPERCONDUCTORS

by

Robert C. Duckworth

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Finally, I would like to thank the Department of Energy for financial support of this project.
This project presents a systematic study of contact resistance and normal zone formation in silver coated YBa$_2$Cu$_3$O$_x$ (YBCO) superconductors. A unique opportunity exists in YBCO superconductors because of the ability to use oxygen annealing to influence the interfacial properties and the planar geometry of this type of superconductor to characterize the contact resistance between the silver and YBCO. The interface represents a region that current must cross when normal zones form in the superconductor and a high contact resistance could impede the current transfer or produce excess Joule heating that would result in premature quench or damage of the sample. While it has been shown in single-crystalline YBCO processing methods that the contact resistance of the silver/YBCO interface can be influenced by post-process oxygen annealing, this has not previously been confirmed for high-density films, nor for samples with complete layers of silver deposited on top of the YBCO. Both the influence of contact resistance and the knowledge of normal zone formation on conductor sized samples is essential for their successful implementation into superconducting applications such as transmission lines and magnets. While normal zone formation and propagation have been studied in other high temperature superconductors, the amount of information with respect to YBCO has been very limited.
This study establishes that the processing method for the YBCO does not affect the contact resistance and mirrors the dependence of contact resistance on oxygen annealing temperature observed in earlier work. It has also been experimentally confirmed that the current transfer length provides an effective representation of the contact resistance when compared to more direct measurements using the traditional four-wire method. Finally for samples with low contact resistance, a combination of experiments and modeling demonstrate an accurate understanding of the key role of silver thickness and substrate thickness on the stability of silver-coated YBCO Rolling Assisted Bi-Axially Textured Substrates conductors. Both the experimental measurements and the one-dimensional model show that increasing the silver thickness results in an increased thermal runaway current; that is, the current above which normal zones continue to grow due to insufficient local cooling.

Approved:

Date

Professor John. M Pfotenhauer
Department of Engineering Physics
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<thead>
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<th>Symbol</th>
<th>Description</th>
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<tr>
<td>A</td>
<td>contact area</td>
<td>$[m^2]$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>volumetric fraction of nickel</td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>volumetric fraction of silver</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>constant</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>specific heat</td>
<td>$[kJ/kg-K]$</td>
</tr>
<tr>
<td>d</td>
<td>thickness</td>
<td>$[m]$</td>
</tr>
<tr>
<td>dx</td>
<td>interval</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>electric field</td>
<td>$[V/m]$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>volumetric fraction of YBCO</td>
<td></td>
</tr>
<tr>
<td>h</td>
<td>heat transfer coefficient</td>
<td>$[W/m^2-K]$</td>
</tr>
<tr>
<td>H</td>
<td>applied external magnetic field</td>
<td>$[T]$</td>
</tr>
<tr>
<td>I,i</td>
<td>current</td>
<td>$[A]$</td>
</tr>
<tr>
<td>j</td>
<td>current density</td>
<td>$[A/cm^2]$</td>
</tr>
<tr>
<td>k</td>
<td>thermal conductivity</td>
<td>$[W/m-K]$</td>
</tr>
<tr>
<td>L</td>
<td>transfer length</td>
<td>$[m]$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>position of defect and edge in TFA YBCO samples with defects</td>
<td>$[m]$</td>
</tr>
<tr>
<td>N</td>
<td>Coefficient to solution of normal metal current in sample with defect</td>
<td>$[A]$</td>
</tr>
<tr>
<td>Q</td>
<td>heat dissipated</td>
<td>$[W]$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>resistivity</td>
<td>$[\Omega-cm]$</td>
</tr>
<tr>
<td>R</td>
<td>interface resistance</td>
<td>$[\Omega-cm^2]$</td>
</tr>
<tr>
<td>T</td>
<td>temperature</td>
<td>$[K]$</td>
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</table>
V voltage $[V]$

w width of sample $[m]$

x position $[m]$

Subscripts

Ag silver

c critical

critical critical at operating temperature

gen generation

i interface

left left of the defect

m matrix

n normal metal

ni nickel

no average normal metal

o ambient

op operating

⊥ perpendicular

right right of the defect

s superconducting

t total cross section

th thermal runaway
Chapter 1

Introduction

1.1 Motivation

Superconductors are materials that carry current without resistance at low temperatures. The first superconductor, mercury, was discovered by the Dutch physicist Kamerlingh Onnes in 1911 [1]. This discovery was aided by his ability to liquefy helium with an atmospheric boiling point of 4.2 K, which he successfully accomplished in 1908. Soon, other metals such as tin and lead were identified to be superconducting at these temperatures. Even at this early stage in their development, Onnes and others saw the vast array of possibilities that wires without resistance held in electrical transmission and magnet construction. However, their optimism was quickly tempered by the inability for early materials to conduct currents equal to those found in standard electrical networks.

In 1933, Walter Meissner and R. Oschsenfeld discovered that if a superconductor is placed in an external magnetic field, currents are established on its surface [1]. These currents create an equal and opposite magnetic field to the one that is being applied and results in a net
zero magnetic field inside the superconductor. This explanation only holds for type I superconductors. It was found later that type II superconductors have two critical fields. The lower critical field, or $H_{c1}$, is the boundary where no field is present in the superconductor and no flux penetration. Above this value and up to the upper critical field, $H_{c2}$, quantized flux exists within the sample, but it still remains superconducting. Outside elements, all chemical compounds are type II. This property allows the superconductors to remain superconducting in high fields, which is advantageous in many applications such as levitation and transformers.

Further research found that the current, applied magnetic field, and temperature are coupled together to define the superconducting limits of a material as shown in figure 1-1. It is these upper limits of critical temperature ($T_c$), critical field ($H_c$), and critical current density ($j_c$) that material scientists and engineers have attempted to improve in the hopes of realizing the applications initially envisioned by Onnes. Prior to 1986, the superconductors that were used in most application were alloys and compounds like NbTi, Nb$_3$Sn, V$_3$Sn, and NbN. These materials were used to construct the first superconducting, high field magnets. However while the field and current carrying properties were satisfactory, critical temperature limited their application. Because the highest critical temperature for these types of compounds is approximately 25 K, liquid helium was preferred because it provided a low enough temperature to maintain the operational requirements of the superconductor. In addition, the alternatives of liquid hydrogen, which required special safety considerations, and cryorefrigerators, which added a level of complexity to operation at that time, were not as attractive. However, the cost of liquid helium, $6/liter, made any application costly to maintain or operate. Recent improvements in refrigeration technology, especially pulse tube refrigerators, have helped in the
After 1986, a new branch of superconductor research blossomed with the discovery of barium lanthanum copper oxide by Alex Mueller and Georg Bednorz, researchers with IBM Zurich [1]. While the critical temperature of 30 K may not seem like much an improvement, it introduced a new material structure, perovskites. Variations of this structure caused the critical temperature of these materials to quickly increase to 90 K and above. Because of the jump in temperature, these superconductors are often referred to as high temperature superconductors (HTS). The increase in critical temperature was significant because now liquid nitrogen, an inexpensive refrigerant ($0.25/liter), could be used instead of liquid helium. While the utopian visions similar to those of Onnes rose as fast as the temperature, processing challenges involving
this ceramic material slowed its transition to commercial conductor production. As these high temperature superconductors are introduced into applications, their properties above the critical region are just as important as the critical properties themselves in understanding and predicting their optimal performance.

When superconducting wires are integrated into electrical systems like transformers or power cables, knowledge of their stability with respect to electrical and thermal disturbances is essential. This information allows engineers to design cryogenic refrigeration systems to withstand these disturbances during operation. \( \text{YBa}_2\text{Cu}_3\text{O}_x \) (YBCO), one of the first HTS materials discovered, is attractive in comparison to other HTS materials such as \( \text{Bi}_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_x \) (Bi-2223) and \( \text{Bi}_2\text{Sr}_2\text{Ca}_1\text{Cu}_2\text{O}_x \) (Bi-2212) due to its higher current densities and their lesser field dependence. Despite difficulties in processing, recent achievements \[2-5\] have produced YBCO coated conductors in long lengths with high critical currents, which are essential in superconducting applications. However before these can be used reliably, it is essential that their behavior in fault conditions and factors that can contribute to their behavior, such as normal zone propagation be fully understood. A key to normal zone state dissipation is the contact resistance between the normal metal and YBCO. A high contact resistance can result in unnecessary Joule heating even when operating currents are less than the critical current. This can lead to premature quench of the sample and limit the amount of current the sample can tolerate. Normal zone formation describes the transition of the superconductor to a resistive state when its critical parameters are exceeded. Knowledge of the formation allows for the construction and operation of stable superconducting devices.
1.2 Scope of Thesis

This thesis deals with issues associated with normal zone formation and contact resistance in silver coated YBCO superconductors. First, the contact resistance is investigated for two different YBCO processing methods to verify whether the dependence of contact resistance on post-annealing oxygen annealing conditions which has been characterized on single crystal substrates translates to more dense, high quality YBCO films. Second, taking advantage of the planar geometry of the YBCO/Ag interface, the correlation between current transfer length and interface contact resistance for a sample with a complete layer of silver is developed and compared to a traditional four-wire measurement method. This provides a better understanding of the role of contact resistance in the stability of YBCO superconductors when defects are present in the material. Finally, a series of silver coated YBCO superconductors are studied with respect to silver thickness and operating currents to define the stability limits of this type of superconductor. These experimental results are compared to a numerical model using a one-dimensional heat equation to better understand the influence of silver and sample substrate thickness on the stability dynamics of this type of superconductor.

1.3 Outline

- A brief overview of the concepts and previous work performed in the areas of contact resistance, normal zone formation, and YBCO materials processing is given Chapter 2. This provides a broad overview of these issues and highlights the lack of information regarding contact resistance and normal zone propagation in silver coated YBCO superconductors for application geometries.
- Chapter 3 begins by describing the current transfer length model, which utilizes the dependence of the voltage profile to calculate the contact resistance. In addition the formulation of a simple one-dimensional model that is used to describe the transient phenomena of normal zones is presented.

- Chapter 4 details the experimental facilities at the University of Wisconsin-Madison and Oak Ridge National Laboratories and the measurement techniques used to characterize the samples.

- The experimental results and their comparison to the models developed in Chapter 3 are given in Chapter 5.

- In Chapter 6, the experimental and theoretical findings are summarized and further areas of research in contact resistance and normal zone formation are given.

- The appendix documents the fortran source code used and the numerical methods and solving techniques of the one-dimensional model described in Chapter 3.
Chapter 2

Review of Literature

2.1 YBCO Processing

2.1.1 YBCO General Information

YBCO was one of the first superconductors to be found with a critical temperature ($T_c$) greater than the atmospheric boiling point of liquid nitrogen temperatures (77 K) [6]. While having similar composition to other thallium (Tl) and bismuth (Bi) superconductors, one of the main differences is in its crystalline structure shown in figure 2-1. YBCO has an orthorhombic structure, which means that the a-axis length is not equal to the b-axis length ($a = 3.83 \, \text{Å}$, $b = 3.88 \, \text{Å}$) [7]. $\text{TlBa}_2\text{Ca}_2\text{Cu}_3\text{O}_x$ (Tl-1223) and $\text{Bi}_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_x$ (Bi-2223) are considered tetragonal with the a-axis length equal to the b-axis length. During the YBCO crystal growth, the difference in these parameters causes twinning, which is a phenomenon where pinning centers are created from the misalignment. Flux flow, which refers to the motion of quantized magnetic vortices within the superconductor in response to an external current, is slowed by the pinning centers that serve as anchors to stop this resistive movement and preserve the superconducting
state. Figure 2-2 shows the benefits of this structure for the orthorhombic YBCO thin films [8] as compared to the tetragonal compounds of epitaxial TlBa\textsubscript{2}Ca\textsubscript{2}Cu\textsubscript{3}O\textsubscript{x} (Tl-1223) on LaAlO\textsubscript{3} [9] and a proton-irradiated Bi\textsubscript{2}Sr\textsubscript{2}Ca\textsubscript{2}Cu\textsubscript{3}O\textsubscript{x} (Bi-2223) powder in tube sample [1]. Despite being able to maintain high current densities in high magnetic fields, processing difficulties have slowed the development of YBCO and Tl-1223 in comparison to Bi-based superconductors. YBCO has received more attention than Tl-1223 because of difficulties in creating mixtures and powders than can be heat treated to produce thallium superconductors successfully.

Figure 2-1. Crystalline structure of YBCO
One issue that must be carefully addressed during processing is the alignment of crystals within the superconducting layer. The YBCO films can be thought as a series of one-dimensional grains of rice pressed into a mold with the grains symbolizing an individual YBCO crystal. As current enters the YBCO, it moves from one adjacent grain to another. If the alignment between adjacent grains is poor, the movement of current through the superconductor is obstructed and the critical current density is low. If the misalignment is kept to a minimum, a high value of $J_c$ is usually achieved.

Controlling the misalignment of crystals represents a challenge for YBCO processing methods. Trifluoroacetates and RABiTs, which are examined in this project, are two methods that achieve high critical current densities. Though both use aligned substrates to minimize the
extent of misalignment in the YBCO, the YBCO is applied to these substrates in two distinct manners.

2.1.2 Trifluoroacetate YBCO thin films

The YBCO thin films provided by American Superconductor are made with a trifluoroacetate process similar to that done by McIntyre [10]. To make this type of YBCO, a precursor solution is prepared initially by dissolving the acetates of yttrium, barium, and copper in a 1:2:3 cation ratio in deionized water. This ratio is determined from the desired stoichiometric quantity of trifluoroacetic acid at room temperature. The LaAlO$_3$ substrates are prepared by first cleaning them ultrasonically and chemically, then by adding enough precursor solution to completely cover the surface of the substrate. The precursor is applied to the substrate at temperatures in the range of 23 °C-31 °C and relative humidities in the range 5 to 40% using a photoresist spinner to minimize contamination of the samples from dust and lint. The solution is dried overnight on square, c-axis oriented LaAlO$_3$ single crystal substrates and typically yields a glassy blue residue.

After drying overnight, the samples are fired in two stages. First, the films are placed in a humid oxygen atmosphere to decompose them to an oxyfluoride material. This is accomplished by heating the sample to 100-120 °C then injecting humid oxygen and heating until a final temperature of 400 °C is achieved. At this point, the sample is allowed to cool in the stagnant humid oxygen atmosphere. The second stage involves annealing the films in a humid nitrogen/oxygen gas mixture at temperatures of 525 °C to convert the oxyfluoride to YBa$_2$Cu$_3$O$_x$. 
The annealing consists of heat treating the sample at a rate of 25 °C/min, and holding at various intervals, until a temperature of 525 °C is reached.

For this particular process, $j_c$ depends on the partial pressure of oxygen in the second stage as well as the thickness of the film itself. An increase in the partial pressure of oxygen from $10^{-4}$ atm to $10^{-1}$ atm results in the decrease in $j_c$ from $3 \times 10^6$ A/cm$^2$ to $1.0 \times 10^6$ A/cm$^2$. An increase of thickness from 50 nm to 300 nm results in an order of magnitude decrease in the critical current density. A scanning electron microscope (SEM) image of a TFA derived sample is shown in figure 2-3 along with an image for a barium fluoride processed YBCO thin film. Both provide a unique geometry on which to deposit the silver and could conceivably affect the YBCO/Ag contact resistance and subsequent stability of the sample.

**Figure 2-3.** Scanning electron microscope images of TFA-derived sample (left) with YBCO thickness of 100 nm. An image of a barium fluoride processed sample with similar thickness (right) is also shown.
2.1.3 Rolling Assisted Bi-Axially Textured Substrate YBCO thin films (RABiTs)

The other YBCO thin films examined were made with the RABiTs processing method that was developed by the materials research group at Oak Ridge National Laboratory [6]. Initially, a polycrystalline, high purity nickel bar is put through rollers to produce nickel deformations greater than 90% and then recrystallized through a vacuum anneal at approximately 400 °C. This produces a nickel substrate with a sharp cube textured surface. The rolling process promotes the alignment of subsequent buffer layers and the $J_c$ enhancement of the overall superconducting film.

The substrates are chosen carefully to avoid interaction with the nickel, which has a tendency to oxidize readily. Initially, this problem can be avoided by depositing a layer of CeO$_2$ onto the nickel substrate under a partial pressure of hydrogen in an oxygen environment. The hydrogen makes the reaction of NiO unstable and allows the CeO$_2$ to be deposited. While the CeO$_2$ prevents the formation of NiO, there exist limitations on its thickness where cracks occur and misalignment begins. So after CeO$_2$ is applied at a thickness of 0.5 µm, yttrium stabilized zirconium (YSZ) is epitaxially deposited to promote the alignment of the (100) direction. The YBCO is finally added to the composite substrate through a barium fluoride process. A SEM image of a typical RABiTS YBCO is given in figure 2-3 and a diagram of the substrate architecture of RABiTs YBCO film is given in figure 2-4.
2.2 Contact Resistance

2.2.1 What is contact resistance?

Contact resistance is the amount of interfacial resistance to the flow of electricity between two adjacent materials and is quantified by the interface resistivity, $R_i$. This parameter is the product of the resistance across the interface and the area through which the current passes and has units of $\Omega\text{-cm}^2$. For YBCO superconductors, a metal layer, usually silver, gold, or platinum, is applied. This metal provides a resistive shunt and allow external current connections to be made without harming the YBCO beneath it. Typically silver is chosen because it has a lower resistance than the other metals and thus can cut down on the heat generation when current is transfer from the YBCO. The creation of a low resistance interface between the YBCO and the metal layer represents an interesting challenge. Unless the metal is
applied in-situ, which is without breaking the vacuum in which the YBCO was grown, the abundance of oxygen near the interface can react with contaminants in the air like water vapor or carbon dioxide to create an insulating layer of material. Without further processing of the sample after the silver is applied, a highly resistive section of material exists between the metal and YBCO and could impede the current flow causing heat generation at the interface. Thus, the selection of the metal and its subsequent application to the YBCO become extremely important.

The phenomenon of contact resistance between YBCO and noble metals, particularly silver (Ag), is of interest to several sectors of the superconducting community. The interface affects both the performance of Josephson junctions and the stability of YBCO thin films and tapes. For Josephson junctions, the application of a silver layer to YBCO allows current to pass into the superconductor without high dissipative losses between the two materials. For thin films and tapes, the use of silver promotes the structural stability of the conductor and allows normal zones that appear in the YBCO to propagate out of the superconductor and dissipate in the matrix material.

**2.2.2 YBCO contact resistance**

After YBCO was discovered in 1987, it was crucial to measure the properties of the YBCO without affecting the quality of the superconductor while minimizing excess Joule heating on the sample. To this end, the smallest interfacial resistivity possible is essential to accurately characterize the properties of YBCO. Prior to the use of Ag/YBCO interfaces, other materials were tested. Ekin [11] investigated the use of indium, silver, and gold as external lead connections and contact pads to YBCO. The lowest observed resistivity of 9 μΩ cm² occurred for a sputtered silver contact pad and gold ribbon wire bonds, while the highest resistivity of
55,000 $\mu\Omega$ cm$^2$ came from indium solder and pads. Further measurements indicated deterioration of the resistivity for the indium contacts when left exposed to air. Suzuki [12] examined the role of sputtered deposition contacts in more detail. He discovered that materials like copper, indium, zinc, and tin showed high resistivities with semiconductive temperature dependence while gold, silver, and platinum have low resistivity with metallic temperature dependence. Suzuki assumed the differences came from an oxygen deficient layer between the metal and YBCO. The role of oxygen was confirmed by Oka [13] as he investigated the vacuum deposition of several metal electrodes (silver, gold, copper, lead, tin, and aluminum) onto bar shaped samples. Both silver and gold contact resistances showed lower contact resistances than other metals. The main difference was thought to lie in the formation of a metal oxide layer between the metal and YBCO layers during processing. For silver and gold, their oxides are unstable and thus the appearance of an oxygen deficient layer on the YBCO interface is difficult. Several groups explored the advantage of silver and gold with YBCO in further detail.

Gold contacts were first examined by Caton [14] who looked for a rugged contact method for sintered YBCO films. This was accomplished through by sandwiching small pieces of metal gold foil by between sintered pieces of YBCO and by reannealing the sample in oxygen to restore the oxygen content at the Au/YBCO interface. Weick [15] pressed pads of Au wire into YBCO powder that was then heat treated to form the YBCO crystals while Jing [16] evaporated a Ag layer on the side of vacuum-cleaved YBCO crystals. Each of these methods produced interface resistivities of $2 \times 10^{-7} \, \Omega\cdot\text{cm}^2$ and $5 \times 10^{-9} \, \Omega\cdot\text{cm}^2$ respectively. Chan et. al. [17] further investigated whether gold would function effectively as a contact to YBCO on a microscopic level. Transmission Electron Microscopy (TEM) images of the interfaces showed a well-bonded
surface with no extraneous phases. The lattice fringes of the (001) plane of the YBCO terminated abruptly at the interface without developing a layer between the two materials. However, silver is preferred because its resistivity is a factor of two less than gold and the oxygen annealing temperature where the contact resistance decreases is smaller (400 °C for silver and 500 °C for gold).

Sun and Yang [18] studied different methods of applying silver to YBCO in order to produce low resistance contacts. Their motivation was to present a convenient and simple technique of applying silver to YBCO without the necessity of sputtering. Painting with a silver powder and subsequently sintering the film at 900 °C produced contact resistances on the order of $1.0 \times 10^{-4} \, \Omega\text{-cm}^2$. Further it was found that higher silver content in the applied film lowered the contact resistance to $1.3 \times 10^{-7} \, \Omega\text{-cm}^2$. However, despite this, Sun and Yang stated that attaching the leads to the silver films was difficult and the upper current limit for these leads was only 5 A.

The majority of research on YBCO contact resistance has centered on either the ex-situ or in-situ deposition of Ag on various films. Ex-situ deposition refers to the process where the sample is transported from the facility where the superconductor is grown to the metal deposition facility, during which process it is exposed to the environment, while in-situ is the process where all the steps are done in one vacuum chamber. Ekin [19] developed a method involving in-situ sputtering of either Ag or Au onto YBCO films after the sample was inserted into a vacuum. The efficiency of in-situ processing was also confirmed by Gong [20]. Ekin’s technique produced 1-6 µm thick contacts when the sample was cooled with water to keep the substrate temperature below 100 °C. The contact resistances of the Ag contacts (10 μΩ-cm²) were on the
order of four magnitudes less than their traditional indium, silver paint, or pressure contact counterparts. The disparity of contact resistance in particular with indium solder was due to the oxide layer that formed between the indium and YBCO. Ekin’s preliminary study and subsequent experimental work found that the contact resistance could be further reduced by completely eliminating air exposure to the sample [21], annealing at an intermediate temperature in oxygen [22], and by optimizing sputter etch and deposition parameters. These studies looked at small, single area contact pads and did not address the case of a layer of silver completely coating the YBCO. Knowledge of the contact resistance in this geometry is essential for understanding the stability of YBCO in the presence of defects and normal zones.

Successful attempts were also made to dope YBCO with silver to improve its contact characteristics. Jin [23] mixed the powder of YBCO and Ag or AgNO₃ before sintering or melt texturing occurred. With In solder pads, he reported contact resistivities of $4 \times 10^{-6} \, \Omega \cdot \text{cm}^2$ for the sintered samples and $4 \times 10^{-8} \, \Omega \cdot \text{cm}^2$ for the melt-textured samples. Similarly, Chen [24] added Ag₂O to $\text{Y}_{0.9}\text{Ca}_{0.1}\text{Ba}_2\text{Cu}_4\text{O}_8$ and found that resistivities of $10^{-8} \, \Omega \, \text{cm}^2$ could be accomplished without the need for annealing with oxygen after the silver content was added. However this material processing produced lengths of only 1 cm long and the amount of time required to produce high current density samples have prevented this processing from being industrially scaled.
2.3 Normal zone formation

2.3.1 What are normal zones?

A normal zone is a resistive area within the superconductor that appears when the local critical parameters of temperature, applied field, or current density have been exceeded. Typically the superconductor reaches these limits when it experiences thermal and/or electrical disturbances. Once a region becomes normal, several things can occur.

- The sample receives sufficient cooling from its surroundings and the normal zone vanishes.
- The normal zone propagates through the superconductor by the conduction of the heat into the metal layer, usually silver or gold.
- Normal zones could form near a defect and not propagate regardless of how little additional cooling is supplied in this case.

The slow or non-existent normal zone propagation can be problematic, with the possibility of causing permanent damage to a magnet coil made out of the superconductor. In the case of low temperature superconductors like Nb₃Sn and NbTi, the normal zone propagation velocity is on the order of 10 m/s and an entire coil can become normal in either a conduction or convection cooling environment in a few seconds – a condition called a magnet quench. In addition the stability margin, which is the amount of energy required to induce normal zones, is typically small at 10’s of mJ/cm³ [25-26]. Since high temperature composite superconductors operate at a higher temperature and therefore have a higher specific heat, the normal zone propagation velocities are typically on the order of 10 cm/s or slower [27-28] and the stability margin
increases to 10 J/cm$^2$ [29]. The slow speed results in a localization of Joule heating, a condition which can lead to thermal runaway and permanent damage in the superconductor [29-30].

2.3.2 Normal zone formation – YBCO

While the majority of work on normal zone propagation in HTS material has focused on Bi-2223, normal zones in YBCO have been examined on a limited basis. Due to the lack of sufficient lengths until recently, most of the effort has concentrated on looking at normal zone formation in fault current limiter (FCL) applications. FCLs rely on a meander-patterned YBCO thin film as shown in figure 2-5 to induce uniform normal zones across the sample length in a short period of time ($\approx$ 10-50 $\mu$s). Vystosky [31] examined normal zone propagation in single and double-sided gold coated YBCO thin films. Double sided refers to the fact that YBCO was grown on both sides of substrate, which was sapphire in this case. The double-sided nature of the sample, which was designed to promote a rapid quench of the entire fault current limiter, allowed for normal zones to appear and disappear through the substrate instead of propagating along the length of the YBCO. Kubota [32] actually observed normal zone propagation velocities on the order of 10-300 cm/s at 77 K in silver coated YBCO fault current limiters. However, the relative width of the YBCO, 0.5 mm, and the silver thickness, 100 nm, makes the implementation of this information on long lengths of tapes difficult. In a similarly patterned YBCO thin films, Kim and Choi [33-34] documented that the appearance of normal zones was influenced by the silver deposition process. Specifically, samples produced with an ex-situ method experience a deterioration in the critical current after repeated quenches at currents much
greater than $I_c$, while those samples produced with an in-situ method showed no change under the same tests.

Chyu and Oberly [35-37] did a series of theoretical studies of a YBCO thin film with a metallic layer for different heat transfer coefficients and operating temperatures. They observed that a higher operating temperature resulted in a higher normal zone propagation velocity, and that an increased heat transfer coefficient increased the minimum energy required to quench the sample. These results however have not yet been confirmed experimentally and did not consider contact resistance.

**Figure 2-5.** Meander pattern of YBCO used in fault current limiter applications.
Chapter 3

THEORETICAL DEVELOPMENT

While the numerical and theoretical study of normal zone formation and propagation has been done in other HTS superconductors like Bi-2223 [38-40], the role of contact resistance and silver thickness in the coated YBCO superconductor geometry have not been addressed experimentally. In addition to its influence on the behavior of normal zones, contact resistance can also affect superconductor stability through defects. Defects that are caused by cracks, impurities, or grain misalignment with the superconductor can force the current into the normal metal at $I < I_c$. Understanding the current transfer in the presence of defects and normal zones is essential for predicting superconductor stability especially if the superconductors are to be integrated into applications.

3.1 Influence of contact resistance in the presence of defects

The role of defects and interface resistivity in coated superconductors was explored theoretically by A.V. Gurevich [41]. This theory was applied to a thin film YBCO sample with a
scratch extending completely across the current path as shown in figure 3-1. Assuming that the interface resistivity is greater than the resistance of the metal on top of the YBCO, or

\[ R_i \gg d_n \rho_n \]  \hspace{1cm} (1)

the current transfer length across interface can be written as

\[ L_i = \frac{R_i d_n}{\rho_n} \]  \hspace{1cm} (2)

As will be shown below, significant information regarding the contact resistance can be obtained from the voltage profile within the cap layer. The voltage distribution within this geometry as a result of the current transfer from the YBCO to the silver cap layer can be found by beginning with Kirchoff’s law and taking the contour integral, C, with respect to the electric field as shown in figure 3-2 to generate an expression for the current density within the normal metal, \( j_n \),

\[ j_n \rho_n dx - R_i [j_\perp(x + dx) - j_\perp(x)] = 0 \]  \hspace{1cm} (3)

**Figure 3-1.** YBCO thin film with defect and illustration of current transfer length associated with the current distribution in silver
Here \( j_\perp \) is the component of the current density flowing normal to the interface surface. This assumes that the time diffusion of the magnetic field within the composite superconductor is slow in comparison to the thermal diffusion. Dividing by \( dx \) and taking the limit \( dx \to 0 \), equation 3 becomes

\[
j_n \rho_n - R_i \frac{\partial j_\perp}{\partial x} = 0 \tag{4}
\]

To find an expression \( j_\perp \) in equation 4, consider a segment of the normal metal of thickness \( dx \) and a height \( d_n \). Assuming conservation of current flow through this segment, the current through the base, \( j_\perp wdx \), is equal to the change in current across the segment in the \( x \)-direction, \([j_n(x+dx) - j_n(x)]wd_n \). Taking the limit as \( dx \to 0 \), we generate the expression

\[
d_n \frac{\partial j_n}{\partial x} = j_\perp \tag{5}
\]

Substituting equation 5 into equation 3, we obtain

**Figure 3-2.** YBCO film with non-conducting buffer layer and substrate with Ag cap layer. Crack at \( x=0 \) across entire width of superconductor.
The solution to equation 6 for a semi-infinite superconductor with a crack dislocation at \( x = 0 \) is

\[
j_n(z) = \frac{i_n}{wd_n} e^{-|x/L_i|}
\]

To find the solution to equation 6 for a sample of finite length, let \( \lambda_1 \) represent the distance from the left edge to the defect and \( \lambda_2 \) represent the total length of the sample. Assuming that \( j_n = i_n/A \) and the cross sectional area, \( A \), is assumed to be constant over the entire length of the sample, the solution to equation 6 for a finite sample is of the form

\[
i_n(x) = A \cosh\left(\frac{x}{L_i}\right) + B \sinh\left(\frac{x}{L_i}\right)
\]

The boundary conditions for this problem are \( i_n(0) = i_n(\lambda_1) = i_n(\lambda_2) = i_o \) where \( i_o \) is the current entering the sample. Applying the boundary equations, the solution breaks down in two portions

\[
i_n(x) = i_o \cosh\left(\frac{x}{L_i}\right) + i_o \frac{1 - \cosh\left(\frac{\lambda_1}{L_i}\right)}{\sinh\left(\frac{\lambda_1}{L_i}\right)} \sinh\left(\frac{x}{L_i}\right) \quad x \leq \lambda_1
\]

\[
i_n(x) = i_o \cosh\left(\frac{x - \lambda_1}{L_i}\right) + i_o \frac{1 - \cosh\left(\frac{\Delta\lambda}{L_i}\right)}{\sinh\left(\frac{\Delta\lambda}{L_i}\right)} \sinh\left(\frac{x - \lambda_1}{L_i}\right) \quad x \geq \lambda_1
\]

where \( \Delta\lambda = \lambda_2 - \lambda_1 \). From these expressions, the voltage as a function of position can be found from the integration of a small differential piece of the normal metal or
Putting equations 9 and 10 into equation 11, the voltage as a function of position takes the form

\[
V = \int_0^z I(x) \frac{\rho dx}{A}
\]  

(11)

where left and right refer the relative position of the integration endpoint with respect to the defect. Two examples of the voltage profiles for differing contact resistances, \(10^{-6} \, \Omega\cdot\text{cm}^2\) and \(10^{-4} \, \Omega\cdot\text{cm}^2\) are shown in figures 3-3 and 3-4 for a silver layer of 1 \(\mu\text{m}\) thick. It is clear from these figures that a contact resistance of less than \(10^{-6} \, \Omega\cdot\text{cm}^2\) will produce voltage profiles of an exponential nature as predicted in the semi-infinite limit and the voltage in the areas to the left and right of the defect is \((\rho_n I/A)/L_i\). For contact resistance values greater than \(10^{-5} \, \Omega\cdot\text{cm}^2\), the functional dependence of the voltage profile must be used to find the current transfer length and subsequently the contact resistance. To obtain the current transfer length, two methods were employed.
Figure 3-3. Voltage as a function of position for a 1 cm long YBCO/Ag sample with a defect at $\lambda_1 = 0.5$ cm, an interface resistance of $10^{-6}$ $\Omega$-cm$^2$, a silver thickness of 1 $\mu$m, and a sample temperature of 77 K.

Figure 3-4. Voltage as a function of position for a 1 cm long YBCO/Ag sample with a defect at $x = 0.5$ cm, an interface resistance of $10^{-4}$ $\Omega$-cm$^2$, a silver thickness of 1 $\mu$m, and a sample temperature of 77 K.
Both techniques involve placing a set of voltage taps far enough away from the current leads to only get the voltage drop across the defect. In the semi-infinite limit, this voltage drop is

\[ V = \frac{\rho_{i} \cdot l}{A} (2L_{i}) \]  \hspace{1cm} (14)

and from equation 2, the contact resistance can be found. The second method takes the relative position of the voltage taps and fits equations 12 and 13 for a given contact resistance.

In order to illustrate the importance of contact resistance with the defects present, the additional heat dissipation from the contact resistance is presented. This can be found from the distribution of the normal metal current density. Assuming the buffer layer and substrate are non-conducting, the total amount of heat dissipated of the composite superconductor is the sum of the dissipation in the Ag layer (n), the interface (i), and the superconductor (s) which can be written as

\[ Q = \int_{-\infty}^{\infty} \left( \rho_{n} j_{n}^{2} d_{n} + R_{i} j_{\perp}^{2} + d_{s} E_{s} j_{s} \right) dx \]  \hspace{1cm} (17)

where it is assumed the conductor is infinitely long in the x-direction. Substituting equation 5 and 7 into equation 17 and integrating with respect to x, the total amount of heat generated becomes

\[ Q = 2w\rho_{n}d_{n}j_{n}^{2}L_{i} \]  \hspace{1cm} (18)

Substituting \( L_{i} \) from equation 2, this reduces further to

\[ Q = 2w j_{n}^{2} d_{n}^{3/2} \sqrt{R_{i}\rho_{n}} \]  \hspace{1cm} (19)

and thus a square root dependence of the heat generation on the interface resistance. For a 5 mm wide conductor with a 1 µm thick YBCO layer, a \( j_{n} \) of 1 MA/cm\(^2\) and interface resistivity of \( R_{i} = \)
$10^{-9} \, \Omega \cdot \text{cm}^2$, the heat generated is about 14 mW for a 1 µm thick Ag cap layer. When a cooling heat flux of 2 W/cm$^2$ is allowed (10% of the pool boiling limit of LN$_2$), an area of 5 x 0.14 mm is needed to maintain the temperature of the sample at 77 K. A value of $R_i = 10^{-5} \, \Omega \cdot \text{cm}^2$ produces 440 mW and a required area of 5.0 x 4.4 mm. While this sounds reasonable, two problems occur. A proximity of defects closer than 4.4 mm would cause a growth of heat generation within the composite superconductor. The second problem appears when tapes are stacked on top of each other and the heat dissipated is transferred out the edges only. Using the same 2 W/cm$^2$ limit, an area of 1 µm x 175 mm is required on either side of the crack to transfer heat into the nitrogen. This length of 175 mm puts an emphasis on the high quality required of the YBCO films across this distance.

### 3.2 Numerical Model

In order to better understand the normal zone formation data that was gathered from experimental measurements, a one-dimensional finite difference model was devised. This was done specifically to examine how changing the silver thickness influences the thermal runaway or quench propagation in the series of RABiTs processed YBCO conductors. Treating the RABiTs superconductor as a single conductor, the time-dependent, non-linear energy equation of

\[
C(T) \frac{dT}{dt} = \frac{\partial}{\partial x} \left( k \frac{dT}{dx} \right) + Q_{\text{gen}}
\]  

\( (20) \)
was solved to determine the temperature profile as a function of position for different silver thickness. The quantities \( k \) and \( C \) represent the volumetrically average thermal conductivity and heat capacity respectively for the conductor including the nickel substrate, or

\[
\begin{align*}
C &= \alpha C_{ni} + \beta C_{Ag} + \gamma C_{YBCO} \\
k &= \alpha k_{ni} + \beta k_{Ag} + \gamma k_{YBCO}
\end{align*}
\] (21)

where \( \alpha, \beta, \) and \( \gamma \) are the volumetric fractions of the nickel, silver, and YBCO respectively.

Assuming the thickness of each layer is uniform, these volumetric fractions reduce to ratios of the thicknesses of each layer. The heat generation term, \( Q_{gen} \), following from the convention used by Iwasa [27] is a product of the current though the entire sample and the voltage drop across the sample, which can be represented as

\[
Q_{gen} = \begin{cases} 
0 & I < I_c \\
\left( \frac{\rho}{A} \right)_m \frac{I}{I_m(T)} I_m(T) & T < T_c, I > I_c \\
\left( \frac{\rho}{A} \right)_m \frac{I^2}{A_t} & T > T_c
\end{cases}
\] (22)

where \( I_m \) represents the current flowing through both the nickel and silver and is the difference between the current and the local critical current when \( T < T_c \) or

\[
I_m = I - I_c(T) = I - I_{co} \left( \frac{T_c - T}{T_c - T_{op}} \right)
\] (23)

where \( I_{co} \) is the critical current at the operating temperature, \( T_{op} \), of 50 K and a critical temperature, \( T_c \), of 90 K is assumed. From the experimental measurements, the nickel and silver
were electrically connected. In order to simulate this effective resistance \((\rho/A)_m\), the nickel and silver were assumed to be in parallel. This leads to the expression for the effective resistance of

\[
\frac{1}{(\rho/A)_m} = \frac{1}{\rho_{ni}/A_{ni}} + \frac{1}{\rho_{ag}/A_{ag}}
\]  

(24)

The boundary conditions that were applied for this model consisted of adiabatic cooling \((Q = 0)\) because the conductor sample is insulated by a layer of Kapton tape from a constant temperature copper block along the entire length of the sample, but has fixed temperatures of 50 K at the ends.

Substituting equations (22) and (23) into equation (20), a second-order implicit Crank-Nicolson method was employed to solve for the temperature distribution using the non-uniform distribution of critical currents obtained from the experiments. From the temperature distribution, the voltage distribution was calculated for each region and compared to experimental results. The thickness of the nickel was used as an adjustable parameter so that the numerical voltage responses were on the same order of magnitude as the corresponding experimental voltages for the same current pulse.

In terms of the physical variables for this system, the adjustable nickel thickness impacts the thermal heat capacity, thermal conductivity, and the effective electrical resistance. For a decrease in nickel thickness from 50 \(\mu\)m to 20 \(\mu\)m, the heat capacity reduces by 5% and the thermal conductivity increases by 10%, while the resistance decreases by 150%. This shows that the nickel thickness directly impacts the effects the electrical resistance while the thermal effects are small in comparison.
Chapter 4

Experimental Setup and Procedures

This chapter describes the experimental arrangement and procedures used to measure contact resistance and normal zone formation in YBCO samples. The influence of contact resistance and silver thickness on normal zone formation was examined using three different techniques to measure the contact resistance. The four wire and current transfer length measurements were applied to the TFA-processed samples, while a hybrid method was used to study the RABiTs-based YBCO samples. In a series of low contact resistance samples, the normal zone formation in YBCO RABiTs samples were examined with respect to silver thickness in a conduction cooling environment.
4.1 Contact Resistance Measurements

4.1.1 Conventional four-wire method

YBCO samples provided by American Superconductor came without silver contacts but with a series of lines etched into the YBCO layer as shown in figure 4-1. This created several areas on one sample all of which received the same oxygen annealing condition. Each of the samples were 1 cm by 1 cm and each etched segment had a width of 2 to 4 mm depending on the number of etched lines on the sample. Typically, the YBCO was 0.35 microns thick and was placed on either a LaAlO$_3$ or YSZ/CeO$_2$ substrate. The critical current densities for these samples are expected to exceed 1 MA/cm$^2$ and the critical current is expected to exceed 5 A. They were on the order of 0.3 to 0.5 MA/cm$^2$ for these samples.

![Figure 4-1. Layout of etched lines (white) on TFA processed samples that were used for four wire technique.](image)

Silver contact pads are applied with a dc sputtering technique and the pad pattern is achieved through laser cut shadow masks as shown in figure 4-2. The masks are made out of...
0.001” stainless steel shim stock and have eight 1 mm² circular holes patterned in two rows of four separated by 3 mm. Both the samples and the masks are placed on a round copper substrate with silver paint to secure them in the vacuum chamber without contamination. This allowed silver contact pads with an area of 1 mm² and thickness between one and six µm to be sputtered onto the YBCO surface. The final silver pattern on the samples is shown in figure 4-3. After the deposition is finished, the samples are placed in an oxygen furnace to anneal. The conditions for the anneal were at an oxygen pressure of 5 psig at a flow rate of 1.0 ft³/min for one hour. The samples are put in a non-reactive YSZ boat in a quartz glass tube to avoid contamination and the annealing is carried out at temperatures between 100 °C and 500 °C.

**Figure 4-2.** Schematic showing the mounting of the sample and mask for silver deposition in TFA-processed YBCO films
After the annealing, gold wires are ultrasonically bonded to the silver surfaces. As noted by Ekin [11], ultrasonically bonded gold wires provide good electrical contacts and produce the lowest interface resistivity without disturbing the silver or the YBCO. The wires are bonded to the silver surface by ultrasonically pushing a gold ball attached to the end of the 0.001” thick wire into the silver interface. The free ends of the gold wire were then soldered to an electronic mount to allow for the external connection of current supplies and voltage taps to the sample. In liquid nitrogen, a single gold wire can carry approximately 0.5 A. With multiple gold wires for the current leads attached on the outer silver pads, currents greater than 2 A can be applied to the sample as shown in figure 4-4. The current was ramped in both the positive and negative directions to eliminate thermal emfs and offsets that affect the actual signal. Current is provided from a HP 6182 DC power supply, while voltages are measured with a Keithley 2182 nanovoltmeter. An example of the voltage current characteristic generated from this technique is shown in figure 4-5. V1 measures the voltage generated by the YBCO itself, which should be
negligible when $I < I_c$. Thus, $V_2$ is the voltage generated from the interface and is twice the contact resistance since the current travels across two silver pads. The interface resistivity is found by multiplying the resistance by the silver pad area.

**Figure 4-4.** Schematic of voltage taps and current lead connections for four-wire technique.

**Figure 4-5.** Voltage as a function of current for $V_1$ and $V_2$ for the four-wire technique.
4.1.2 Current transfer length method

Using another set of TFA-processed samples from American Superconductor, this technique utilizes the correlation in equation 2 from section 3.1 to determine the contact resistance. To create a situation where the current transfer length can be measured, a scratch was introduced across the entire width of the substrate. As the YBCO is grown, an area around the scratch of low \( T_c \) and \( I_c \) is created and a deficient region is visible as a line across the YBCO. A complete layer of silver is deposited on top of the YBCO sample at the University of Wisconsin. This allows the current to flow into the silver when the critical parameters are exceeded. With a series of voltage taps as shown in figure 4-6, the two methods that were discussed in section 3.1 were carried out. Initially differential measurements were taken by feeding a bipolar current through the sample and measuring the voltages to establish the regions in the material that are superconducting and the critical current at each temperature. Then a series of voltage taps referenced to a single tap on the right provided a voltage profile from which the current transfer length was measured. An example of the voltage profile is shown in figure 4-7. With this and the differential voltage measurements across the defect, the contact resistance was found. This method is compared to the four-wire method results (see figures 5-1 and 5-2) and those of Ekin [22] and demonstrates that the current transfer length formulation of contact resistance produces the same values.
**Figure 4-6.** Schematic of voltage taps and current leads on silver coated YBCO sample with defect used to measure the current transfer length.

**Figure 4-7.** Example of voltage profile in sample with defect.
Due to a limited number of samples, multiple regions of silver coated YBCO were created on one sample. To accomplish this, a complete layer silver of 1 \(\mu\text{m}\) thick was deposited across the sample. Using a mask to cover half of the sample perpendicular to the defect, another 2 \(\mu\text{m}\) of silver was deposited. After a laser cuts down to the sample substrate perpendicular to the defect, two areas with a thickness of 1 \(\mu\text{m}\) and two areas with thickness of 3 \(\mu\text{m}\) were created as shown in figure 4-8.

**Figure 4-8.** Geometry of silver layers in a sample used to make multiple measurements of the current transfer length of the YBCO/silver interface on one sample.
4.1.3 ORNL contact method

A flexible and portable method has been devised to measure the contact resistance in non-defect silver coated samples fabricated at ORNL. The method is similar to the four wire method in that it used the contact resistance generated through a given current contact area to get the interface resistivity. A hybrid current lead / voltage tap was constructed as shown in figure 4-9 with a spring loaded pin inserted into a copper current lead of fixed area. With a series of

![Diagram of hybrid voltage - current probes](image)

**Figure 4-9.** Hybrid voltage – current probes used to measure the contact resistance for the RABiTs-processed samples at Oak Ridge National Laboratories.

four probes secured to the sample, the outer probes were attached to the power supply to provide current while those not connected were used to monitor the superconducting state of the YBCO in a manner similar to that shown in figure 4-4. The gap between the current lead and voltage
tap controls the resolution of this method. With a gap of 50 \( \mu \text{m} \) and a silver thickness of 1 \( \mu \text{m} \), a resolution of \( 10^{-7} \Omega \text{-cm}^2 \) for the contact resistance is achieved.

### 4.2 Normal Zone Formation

For RABiTs-processed YBCO samples, the technique used to characterize the normal zone formation consists of a series of equally spaced voltage taps that divide the sample into several regions. This is done to better determine the uniformity of the sample and identify regions of low critical current. A schematic of the normal zone formation setup is given in figure 4-10. Once a given operating temperature was established, the critical currents were determined by rapidly increasing and decreasing the current through a given maximum and using the standard 1 \( \mu \text{V/cm} \) criteria. Allowing the sample to return to thermal equilibrium, a two stage current waveform as pictured in figure 4-11 is applied to the sample. The pulse current is used to

![Figure 4-10. Schematic of normal zone formation measurements](image)
induce normal zones in the sample and examine the influence of the initial energy by varying the pulse magnitude and duration. The secondary operating current is used to monitor sample quench or recovery. A data acquisition system synchronized with the power supply by an output voltage is used to sample the voltage signals across the sample. This procedure is repeated for different pulse and operating current magnitudes and pulse durations to determine the impact of contact resistance and silver thickness on stability.

![Figure 4-11. Time dependence of two stage current waveform that was applied to sample to characterize normal zone formation. These values were set above the lowest $I_c$ in the sample and the time scale of this waveform was between 5 and 40 s.](image)

4.3 Experimental Setups

4.3.1 UW-ASC facilities

The contact resistance of the TFA-processed films was measured at different temperatures between 40 K and 77 K. Liquid nitrogen (77 K) provided a robust, convective cooling environment to characterize the quality of the samples without permanently altering their properties. After the measurements at 77 K were completed, an AL60 Cryomech refrigerator was used to provide a conduction-cooling environment. This single stage GM cryocooler, which is shown in figures 4-12 and 4-13, was placed in a standard liquid helium dewar and the space
surrounding the cold head was evacuated to $10^{-3}$ Torr. Through the use of a liquid nitrogen jacket and an outer copper shield to cut down on the radiation heat leak into the system, temperatures between 40 K and 70 K were achieved.

![Figure 4-12. Picture of AL60 cryocooler setup for the measurement of contact resistance and normal zone formation](image)
Figure 4-13. Schematic of copper radiation shield and sample holder for mounting to the AL60 cryocooler.

4.3.2 ORNL facilities

Like the UW-ASC facilities, a liquid nitrogen bath is used to characterize the sample initially. Samples of lengths between 5 and 20 centimeters are attached to copper plate that is thermally anchored to the 1st stage of a Cryomech GB37 cryocooler (figures 4-14 and 4-15). The cold end temperatures for this cryocooler with no heat load are 10 K for the 2nd stage and 40 K for the 1st stage. The current leads are secured to the first stage which has a higher cooling capacity and allowed for currents up to 200 A to be applied to the sample without significant impact on the sample temperature. The placement of the samples on the bottom plate is two fold. First, limited mounting space and equipment restricts samples to this length on the bottom copper plate. Second, the copper plate at the bottom provides a large isothermal heat sink where
the temperature is near 50 K. With the sample secured to a Kapton-insulated copper block, a steady state temperature of 50 K is reached in approximately 8 hours.

**Figure 4-14.** Picture of GB37 Cryomech cryocooler used for the normal zone formation characterizations.
Figure 4-15. Schematic of GB37 Cryomech cryocooler used for the normal zone formation characterizations. The sample is thermal anchored to the 1st stage of the cryocooler through the copper pipes that run between the 1st stage of the cryocooler and the copper support plate.
Chapter 5

Results and Analysis

5.1 Contact Resistance Measurements

5.1.1 Four Wire Measurements

A series of silver-patterned YBCO thin films were oxygen annealed at temperatures between 100 °C and 450 °C for one hour. After multiple gold wires were attached to the silver pads to provide current and voltage connections, the voltage drops across the sample and the silver/YBCO interfaces were measured at 77 K. Figures 5-1 and 5-2 show examples of the current-voltage characteristics observed for these samples at 77 K. Since no voltage was observed for V1 from figures 5-1 and 5-2, the sample was assumed to be in the superconducting state for those currents and the voltage in V2 is generated from the resistance of the interface and the silver pads. For the silver at 77 K, the resistance for current contact pads with a silver thickness of 1 μm and area of 1 mm is 5.3 nΩ, which is significantly less than the resistances seen in figures 5-1 and 5-2. The contact resistance was
Figure 5-1. Current-voltage characteristics for V1 and V2 for sample tfa1559, which was oxygen annealed at 100 °C for one hour. The slope of 0.0475 Ω gives a contact resistance of $1.86 \times 10^{-4} \Omega \cdot \text{cm}^2$ at 77 K.

Figure 5-2. Current-voltage characteristics for V1 and V2 for sample tfa1561, which was oxygen annealed at 425 °C for one hour. The slope of 12 µΩ gives a contact resistance of $4.30 \times 10^{-8} \Omega \cdot \text{cm}^2$ at 77 K.
obtained by multiplying the measured resistance by the area of the silver pad and divide by two, or

\[ \rho_i = \frac{RA}{2} \]

Figure 5-3 compares the composite data of the contact resistance as a function of oxygen annealing temperature with results obtained by Ekin [22] for ex-situ, silver coated pulsed laser deposited YBCO. The agreement between the experimental results and the previous work indicate that the role of the YBCO processing method in the contact resistance is minimal.

![Figure 5-3](image-url)

**Figure 5-3.** Contact resistance at 77 K as a function of oxygen annealing temperature for the TFA-processed films. Data from Ekin [22] is provided for comparison.

One factor that does play a role in the measurement of contact resistance is the relative location of the voltage taps to current leads on the current carrying silver pads. If the voltage pad is at a distance greater than \( L_i \) as shown in figure 5-4, the voltage tap no longer measures the
voltage drop across the interface. Figure 5-5 shows the result from the placement of the voltage tap outside of this distance. This affects the linearity of the curve from its relative location with respect to the current source and subsequently gives an incorrect measurement of the contact resistance.

Figure 5-4. Typical geometry of the current lead and voltage taps for ultrasonically bonded gold wires attached to the silver pad. The distance, L_i, represent the distance at which the voltage drop is no longer due to the interface and the contact resistance can not be measured.

Figure 5-5. Comparison of the voltage-current characteristics when the voltage is and is not within a distance L_i of the current lead
5.1.2 Current Transfer Length Measurements

Two samples with defects across the entire width of the sample were patterned with the two-tier silver thickness structure shown in figure 4-8. Each sample has two sections with silver thicknesses of 1 µm and 3 µm. Sample tfa2011 was oxygen annealed for one hour at 450 °C and sample tfa2012 was oxygen annealed for one hour at 250 °C. Expecting similar critical currents for both samples (2-3 A) as seen in other TFA-processed samples, initial measurements were taken in liquid nitrogen (77 K). Voltage profiles for each silver thickness in each sample are shown in figures 5-6 through 5-9. For sample tfa2011, the contact resistance was expected to be near $10^{-8} \, \Omega \cdot \text{cm}^2$, which meant that the voltage on either side of the defect should be constant as shown in figure 3-3. The linear nature of the slope however indicates current sharing between

![](image)

**Figure 5-6.** Voltage profile for sample tfa2011, which was annealed at 450 °C, with a silver thickness of 1 µm at a current of 100 mA at 77 K. Assuming a shared current of 12 mA, the curve fit shown above, which was generated from equations 12 and 13, produced contact resistance of $1.00 \times 10^{-8} \, \Omega \cdot \text{cm}^2$. 
Figure 5-7. Voltage profile for sample tfa2011, which was annealed at 450 °C, with a silver thickness of 3 μm at a current of 100 mA at 77 K. Assuming a shared current of 10 mA, a contact resistance of $6.00 \times 10^{-8} \Omega \text{cm}^2$ was found from equations 12 and 13.

Figure 5-8. Voltage profile for sample tfa2012, which was annealed at 250 °C, with a silver thickness of 1 μm at a current of 100 mA at 77 K. Assuming a shared current of 5 mA, a contact resistance of $5.60 \times 10^{-7} \Omega \text{cm}^2$ was found from equations 12 and 13.
Figure 5-9. Voltage profile for sample tfa2012, which was annealed at 250 °C, with a silver thickness of 3 µm at a current of 100 mA at 77 K. Assuming a shared current of 14 mA, a contact resistance of $7.75 \times 10^{-6} \, \Omega \cdot \text{cm}^2$ was found from equations 12 and 13.

YBCO and silver. This was the result of an operating current greater than the critical current applied to the sample. This current sharing adds a linear position dependent term to equations 12 and 13. To obtain the current transfer length and the contact resistance, the percentage of the current sharing in the silver across the entire sample length and the current transfer length were varied to obtain an accurate curve fit for each silver thickness. In this test, the minimum operating current was 0.01 mA and because of this low current, the voltage generated was smaller than the resolution of the Keithley nanovoltmeter ($< 10 \, \text{nV}$). The curve fits are displayed in figures 5-6 through 5-9. The contact resistances obtained from these figures are displayed in figure 5-10 as a function of annealing temperature along with the four-wire measurements and the previous values of Ekin [22]. The current transfer length method comes
closer to the Ekin’s values of contact resistance than the four wire method while showing the same dependence on oxygen annealing temperature.

![Graph showing contact resistance vs. oxygen annealing temperature]

**Figure 5-10.** Comparison of contact resistance found from the current transfer length method to four-wire measurements and Ekin data [22] at 77 K.

In order to eliminate the influence of the normal state on the voltage profile, the sample temperature was lowered. This would increase the critical current and allow for the voltage profile to reflect the contribution from the current transfer across the YBCO/Ag interface.

Figure 5-11 and 5-12 show the four-wire measurement and current transfer length measurement
Figure 5-11. Voltage as a function of current at 40 K for sample tfa1561, which was annealed at 450 °C and had a silver thickness of 1 μm. This four-wire measurement produced a slope of $6.557 \times 10^{-6} \Omega$, which gives a contact resistance of $1.23 \times 10^{-8} \Omega \cdot \text{cm}^2$.

Figure 5-12. Voltage profile for sample tfa2011 at 40 K and a silver thickness of 1 μm at 100 mA from current transfer length method. The differential voltage drop across the defect results in a contact resistance of $1.38 \times 10^{-8} \Omega \cdot \text{cm}^2$. The line represents the voltage profile generated from equations 12 and 13 for the same contact resistance.
respectively of samples that were annealed at 450 °C and measured at a sample temperature of 40 K. The limitation on the number of points in figure 5-11 was due to the gold wire leads. In liquid nitrogen, each 0.001” thick gold wire could tolerate 0.7 A before burning out. In the vacuum environment of 40 K, any current slightly above 0.1 A caused repeated failure of the current leads. Both methods produce similar contact resistances of $1.28 \times 10^{-8} \ \Omega\cdot\text{cm}^2$ and $1.38 \times 10^{-8} \ \Omega\cdot\text{cm}^2$. An example of the voltage profile for sample tfa2012 at 40 K is shown in figure 5-13 for the case when the contact resistance is greater than $10^{-6} \ \Omega\cdot\text{cm}^2$. This supports the current transfer correlation for contact resistance and confirms the ability of a defect to transfer current completely to the silver layer. Since applications deal in the temperature range between 40 K and 77 K, the contact resistance was measured for sample tfa2011 and is shown in in figure 5-14.

![Figure 5-13](image-url)  
Figure 5-13. Voltage profile for sample tfa2012 at 40 K with a silver thickness of 3 μm at 100 mA from the current transfer length method. A contact resistance of $3.160 \times 10^{-6} \ \Omega\cdot\text{cm}^2$ was found from equations 12 and 13.
Figure 5-14. Contact resistance at different temperatures and different silver thickness for sample tfa2011. This sample was oxygen annealed at a temperature of 450 °C for one hour.

5.1.3 ORNL Contact Resistance Measurements

Figure 5-15 shows the voltage as a function of current for a RABiTS-based YBCO sample that was annealed at 500 °C. Since no voltage was seen for V1, the contact resistance is less than the $10^{-7} \ \Omega \cdot \text{cm}^2$ resolution for this method. This agrees with Ekin’s results, which produced a contact resistance on order of $10^{-9} \ \Omega \cdot \text{cm}^2$ at this oxygen annealing temperature.
5.2 Normal Zone Formation

5.2.1 Influence of silver thickness – Experimental Results

To characterize normal zone formation in silver-coated YBCO superconductors, the critical current profile along the length of the sample was measured. By dividing the sample with a series of voltage taps as shown in figure 5-16, the regions of lowest critical currents are isolated and the origin and formation of normal zones in the samples is better understood. Using the GB37 cryocooler to control the operating temperature at 50 K and the standard voltage criteria of 1 µV/cm to define critical currents, the local critical currents were found at 50 K for a set of three RABiTS-processed samples with silver thicknesses of 2 µm, 4 µm, and 8 µm. These
values are shown in Table 5-1. The decrease in critical currents that is observed in samples 2 (4 \( \mu \text{m} \)) and 3 (8 \( \mu \text{m} \)) is related to the materials processing that they experienced before measurements were made. Initially the quality of samples 2 and 3 was analyzed using an x-ray diffraction technique at Oak Ridge National Laboratories, which judges the texture of the sample. This x-ray technique requires a 1.5 \( \mu \text{m} \) layer of silver to be deposited first to look at the critical currents. Then this silver layer is chemically etched off the sample until only the YBCO remains. During this chemical processing, the overall thickness of the YBCO was reduced. Assuming the quality of the sample is consistent, the critical current density is constant along the thickness of the YBCO, thus the critical current, \( I_c \), is reduced since \( I_c \) is the product of the critical current density and the cross sectional area of the YBCO.
After the critical currents were determined, the two-stage current waveform was applied to the sample as shown previously in figure 4-11. The first stage of the current pulse is a large amplitude, short duration pulse between 0.5 and 2.0 s intended to induce normal zones across the entire length of the samples. The second stage of the current pulse, the operating current, is of lower amplitude but long duration typically between 5.0 and 40.0 s and is used to monitor the normal zone development. In these tests both the first and second stage characteristics of the current were varied to find the limits of the thermal runaway currents. The thermal runaway current, $I_{th}$, is defined as the current that produces sufficient heating to exceed the local cooling and cause a temperature rise associated with a steady increase in voltage to occur. With a fixed current pulse inducing normal zones, the operating current was started slightly above the lowest value of $I_c$ in the sample. In successive tests, the operating current was increased until the measured voltages began to increase with time during the operating current. Figures 5-17 and 5-18 show examples that illustrate recovery for this set of samples, while figures 5-19 and 5-20 are cases where thermal runaway occurred.

**TABLE 5-1.** Critical currents at 50 K of samples 1, 2, and 3 with silver thicknesses of 2 µm, 4 µm, and 8 µm respectively.

<table>
<thead>
<tr>
<th>Voltage Region</th>
<th>Sample 1 $d_n = 2$ µm $I_c$ [A]</th>
<th>Sample 2 $d_n = 4$ µm $I_c$ [A]</th>
<th>Sample 3 $d_n = 8$ µm $I_c$ [A]</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>12.5</td>
<td>5.78</td>
<td>4.62</td>
</tr>
<tr>
<td>V2</td>
<td>16.1</td>
<td>8.38</td>
<td>8.27</td>
</tr>
<tr>
<td>V3</td>
<td>13.0</td>
<td>6.86</td>
<td>6.24</td>
</tr>
<tr>
<td>V4</td>
<td>12.2</td>
<td>5.03</td>
<td>4.47</td>
</tr>
<tr>
<td>V5</td>
<td>18.0</td>
<td>5.53</td>
<td>5.82</td>
</tr>
<tr>
<td>V6</td>
<td>28.1</td>
<td>5.11</td>
<td>3.78</td>
</tr>
<tr>
<td>V7</td>
<td>22.0</td>
<td>3.33</td>
<td>3.38</td>
</tr>
<tr>
<td>V8</td>
<td>21.9</td>
<td>4.11</td>
<td>2.73</td>
</tr>
</tbody>
</table>
Figure 5-17. Recovery as seen in sample 2 when it experienced a pulse current of 56.0 A for 0.5 s followed by an operating current of 6.0 A at 50 K. The voltage regions of V1, V2, and V3 are the same as those shown in figure 5-16 and have critical currents of 5.78 A, 8.38 A, and 6.86 A respectively.

Figure 5-18. Recovery as seen in sample 3 when it experienced a pulse current of 71.5 A for 0.5 s followed by an operating current of 12.9 A at 50 K. The voltage regions of V1, V2, and V3 are the same as those shown in figure 5-16 and have critical currents of 4.62 A, 8.27 A, and 6.24 A respectively.
Figure 5-19. Thermal runaway as seen in sample 1 when it experienced a pulse current of 37.5 A for 0.5 s followed by an operating current of 14.0 A at 50 K. The voltage regions of V1, V2, and V3 correspond to those shown in figure 5-16 and have critical currents of 12.5 A, 16.1 A, and 13.0 A respectively.

Figure 5-20. Thermal runaway as seen in sample 2 when it experienced a pulse current of 22.0 A for 0.5 s followed by an operating current of 14.4 A at 50 K. The voltage regions of V1, V2, and V3 correspond to those shown in figure 5-16 and have critical currents of 5.78 A, 8.38 A, and 6.86 A respectively.
It was found that the thermal runaway current was independent of the initial current pulse, which explains the difference in current pulses for figures 5-18 and 5-20.

Since the critical currents for each sample were different, the value of thermal runaway current was scaled to the lowest region of critical current in the sample to better understand the influence of silver thickness. The values of the thermal runaway currents and their scaled values are shown in figures 5-21 and 5-22.

Figure 5-21. Experimental thermal runaway currents as a function of silver thickness at a sample temperature of 50 K.
Figure 5-23. Experimental thermal runaway currents scaled to the lowest critical current of each sample as a function of silver thickness at a sample temperature of 50 K.

The pulse current magnitude and duration were also varied to see if the initial energy supplied to the sample had any effect on its stability. Figure 5-23 represents a case when the operating current was greater than $I_c$ for a variety of different current pulses. In this case, we find that increasing the thermal disturbance does not impact the recovery in these samples. Despite thermal disturbances up to 800 J/cm$^3$, which were found by integrating the voltages over the time during the current pulse, the thermal runaway current was not affected. This demonstrates the durability of the sample under short high current pulses.
5.2.2. Influence of silver thickness – Numerical Results

The numerical model described in section 3.2 has been applied to the conditions used in the experimental tests described in the previous section to better understand the quench dynamics. To provide an accurate characterization, one individual recovery run was selected for each sample and the nickel thickness was adjusted until the calculated voltages during the pulse current matched the magnitude of those seen in the experimental results. As previously mentioned, the nickel thickness controlled the electrical resistance and was fixed during the remainder of the calculations for each sample. Figures 5-24 and 5-25 show a direct comparison of the experimental results and the numerical calculations when the sample experiences recovery for the same pulse and operating currents. A similar direct comparison is shown in figures 5-26.
Figure 5-24. Experimental voltages as a function of time for sample 3 when it experienced a pulse current of 41.4 A for 0.5 s followed by an operating current of 13.6 A at a sample temperature of 50 K.

Figure 5-25. Numerical voltages as a function of time for sample 3 when it experienced a pulse current of 41.4 A for 0.5 s followed by an operating current of 13.6 A at a sample temperature of 50 K.
and 5-27 for the case of thermal runaway. It was found from figure 5-27 that runaway did not happen at the same current numerically as it did experimentally. Since the assumptions of the numerical boundary conditions and complete uniform contact between the nickel and silver might be slightly different than in the experimental case, the current was increased in the numerical calculations until runaway was observed as shown in figure 5-28. An experimental equivalent to figure 5-28 was not done because the availability of the samples is extremely limited and preservation of the sample was important.

**Figure 5-26.** Experimental voltages as a function of time for sample 3 when it experienced a pulse current of 41.4 A for 0.5 s followed by an operating current of 15.8 A at a sample temperature of 50 K. The arrow indicates the voltages during the current pulse are greater than 0.001 V and extend beyond the range of the graph.
Figure 5-27. Numerical voltages as a function of time for sample 3 when it experienced a pulse current of 41.4 A for 0.5 s followed by an operating current of 15.8 A at a sample temperature of 50 K. The arrow indicates the voltages during the current pulse are greater than 0.002 V and extend beyond the range of the graph.

Figure 5-28. Numerical voltages as a function of time for sample 3 when it experienced a pulse current of 41.4 A for 0.5 s followed by an operating current of 20.5 A at a sample temperature of 50 K.
To draw an accurate characterization of the thermal runaway current between each sample, the absolute value of the thermal runaway current, $I_{th}$, and the scaled thermal runaway current, $I_{th}/I_c$, are shown in figures 5-29 and 5-30. A clear dependence of the silver thickness is supported, but the difference between the absolute values of the thermal runaway current suggest an additional effect. For sample 1, the thermal runaway current is identified based on the signal from section V1 as shown in figure 5-19. The increased resistance in this section could be the result of either a reduction in silver thickness or poor contact to the nickel. This would lead to an increase in the joule heating and causes thermal runaway. This indicates that nickel might aid lowering the resistance of the sample when normal zones are present and take a more active role in the quench dynamics of this type of superconductors. It appears to be an area for further investigation.

![Figure 5-29. Numerical and experimental thermal runaway currents, $I_{th}$, as a function of silver thickness at a sample temperature of 50 K.](image)
Figure 5-30. Numerical and experimental thermal runaway currents scaled to the lowest region of critical current, $I_{th}/I_c$, as a function of silver thickness at a sample temperature of 50 K.
Chapter 6

Conclusions

The roles of contact resistance between YBCO and silver, and the silver thickness in the quench dynamics of silver-coated YBCO superconductors have been studied. This investigation has confirmed the consistency of determining contact resistance using two different approaches. The current transfer length method was found to be equivalent to the four-wire method in characterizing contact resistance of the YBCO/Ag interface at different temperatures. This method has shown the general dependence of the contact resistance on oxygen annealing temperatures and established that the YBCO processing method does not influence the contact resistance. This investigation has also explored the influence of silver and substrate thickness in quench dynamics. The numerical and experimental studies on the RABiTs superconductors of different silver thickness have shown that the silver thickness and its contact to the nickel substrate play a tremendous role in the stability on the superconductors in a cryocooled environment.
The four-wire measurements of the contact resistance in TFA-processed and RABiTS-processed YBCO samples mirror the dependence on oxygen annealing temperature with temperatures above 400 °C producing contact resistances less than $10^{-8} \, \Omega \cdot \text{cm}^2$. The dependence on oxygen annealing temperature was also observed in the current transfer length method, which validates its characterization of the YBCO/Ag interface and its contact resistance. The influence of silver thickness on contact resistance that was found using the current transfer length method indicated a possible change in the silver diffusion dynamics as the silver thickness increased.

The experimental results and numerical calculations indicate the importance of silver in the response of silver-coated YBCO superconductors to electrical disturbances. Increasing the silver thickness from 2 µm to 4 µm caused a factor of two increase in thermal runaway current when rationalized to the lowest region of critical current. Further exploration with the numerical model that was based on the one-dimensional heat equation for the bulk sample suggests that contact to the nickel substrate allows the sample to withstand high currents due to the lower effective resistance. In addition, the thickness of the nickel provides increased thermal ballast so that the conductor can withstand large disturbances.

Future efforts should be directed toward the measurements of the contact resistance using the current transfer length method on higher quality and longer length samples. Since normal zones were presented at 77 K for our current transfer length measurements, higher quality samples would eliminate this contribution to the voltage signal and allow for the current transfer length and the contact resistance to be readily identified. Longer length samples would also allow for more voltage taps and better definition of the current transfer length at low oxygen
annealing temperatures. Longer lengths would also allow for multiple defects to be added to the system and the uniformity of the contact resistance across the sample to be characterized. Since the current transfer length for contact resistances greater than $10^{-4} \ \Omega \cdot \text{cm}^2$ produce a linear profile for 1 cm lengths, longer lengths would also expand the range of oxygen annealing temperature that can be studied using this method. The dependence of silver thickness should also be systematically studied through both voltage measurements and imaging of the interface to correlate thickness to the contact resistance measured and quality of the interface. In terms of stability, two main issues need further investigation. First, the influence of contact resistance on normal zone formation should be established to determine the oxygen annealing conditions required to maintain safe operation of these types of superconductors. Second, the importance of the electrical contact between nickel and silver in these studies indicate a need to examine the influence of the substrate material and thickness on normal zone formation and propagation.
Reference:


40. Bellis, R.H. and Iwasa, Y., “Quench propagation in high T_c superconductors”, Cryogenics (1994), 34, p. 129


Appendix A

Computational Model

In this appendix, a detailed description is given of the computational methods used to model the one-dimensional heat equation

$$C(T) \frac{dT}{dt} = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + Q_{\text{gen}} \quad (A.1)$$

Initially a series of \( n \) uniformly spaced temperature nodes were created along the length of the sample as shown in figure A-1. These nodes were grouped into 10 sections, eight corresponding to the voltage tap zones established in the experiment and two additional zones added to include the current leads in the boundary conditions of the thermal calculation.

For the current distribution, it was assumed that only when the critical current was exceeded, current was in silver and nickel matrix. This value was calculated as

$$I_m[i] = I - I_c(T[i]) = I - I_{co}[i] \left( \frac{T_c - T[i]}{T_c - T_{op}} \right) \quad \text{I} > I_c[i] \quad (A.2)$$
Figure A-1. Distribution of nodes along length of sample. Shows division of sample into 10 distinct 2 cm zones and an enlargement of the energy balance on the ith node.

where the subscript i refers a specific node in the distribution where the temperature and critical current at the initial operating temperature are defined. After the current distribution was found, a second order Crank-Nicolson finite difference method was used to incorporate the new temperature at $T_{\text{new}}[i]$ with the previous temperature $T[i]$. This creates the following expression for each term of in equation (A.1)

\[
C(T) \frac{dT}{dt} = C(T[i]) \frac{T_{\text{new}}[i] - T[i]}{dt}
\]  

\[\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) = \frac{k(T[i])}{2} \left[ \frac{T_{\text{new}}[i + 1] - 2T_{\text{new}}[i] + T_{\text{new}}[i - 1]}{dx^2} \right] + \left( \frac{T[i + 1] - 2T[i] + T[i - 1]}{dx^2} \right) \]

(A.4)
As mentioned in chapter 3, the thermal properties of thermal conductivity and heat capacity are volumetrically averaged over the entire sample section and like the effective resistivity is evaluated at $T[i]$ to simplify the numerical model. In order to solve the heat equation numerically, the terms with $T_{\text{new}}[i]$ were grouped and moved to the left hand side of the equation A.1 and $T[i]$ terms were grouped on the right hand side of the equation A.1 to produce

$$a[i] \cdot T_{\text{new}}[i + 1] + b[i] \cdot T_{\text{new}}[i] + c[i] \cdot T_{\text{new}}[i - 1] = rts[i]$$

(A.6)

where

$$a[i] = -\frac{k}{2dx^2}$$

(A.7)

$$b[i] = \frac{C(T[i])}{dt} + \frac{k}{dx^2}$$

for $T[i] > T_c$ or $I < I_c[i]$  

(A.8a)

$$b[i] = \frac{C(T[i])}{dt} + \frac{k}{dx^2} - \left(\frac{\rho}{A} \right) \frac{I}{A_i} \frac{T_{\text{co}}[i]}{T_c - T_{\text{op}}}$$

for $T[i] < T_c$ and $I > I_c[i]$  

(A.8b)

$$c[i] = -\frac{k}{2dx^2}$$

(A.9)

$$rts[i] = \frac{C(T[i])}{dt} + \frac{k}{2dx^2} \left(T[i + 1] - 2T[i] + T[i - 1]\right)$$

for $I < I_c[i]$  

(A.10a)
\[
rts[i] = \frac{C(T[i])}{dt} + \frac{k}{2dx^2} (T[i+1] - 2T[i] + T[i-1]) + \left( \frac{\rho}{A} \right) \frac{I}{A_i} \left( I - \frac{I_{co[i]}T_c}{T_c - T_{op}} + \frac{I_{co[i]}T[i]}{2(T_c - T_{op})} \right)
\]
for \( I > I_c[i], T < T_c[i] \) \hspace{1cm} (A.10b)

\[
rts[i] = \frac{C(T[i])}{dt} + \frac{k}{2dx^2} (T[i+1] - 2T[i] + T[i-1]) + \left( \frac{\rho}{A} \right) \frac{I^2}{A_i} \text{ for } T > T_c[i]
\]
(A.10c)

When equation A.6 is extended across the entire length of the sample, a tri-diagonal matrix is created as shown below.

\[
\begin{bmatrix}
b[1] & c[1] & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & 0 & a[i] & b[i] & c[i] & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & 0 & 0 & 0 & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & 0 & 0 & 0 & 0 & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & a[n] & b[n] & c[n] \\
\end{bmatrix}
\begin{bmatrix}
Tnew[1] \\
\vdots \\
\vdots \\
\vdots \\
Tnew[i] \\
\vdots \\
\vdots \\
\vdots \\
Tnew[n] \\
\end{bmatrix}
= \begin{bmatrix}
rts[1] \\
\vdots \\
\vdots \\
\vdots \\
rts[i] \\
\vdots \\
\vdots \\
\vdots \\
rts[n] \\
\end{bmatrix}
\]

These sets of equations are solved with the Thomas algorithm to find the updated temperature profile at t+dt. The values are then substituted into the temperature dependent equations for the electrical and physical properties and these are used to recalculate the current distribution and the temperature profile at next iteration step. During each time step, the voltages are found by taking the temperature and calculating the resistances over each 2 cm section the nickel and silver. These resistances are then used to calculate the current through the silver and then the voltage that would be measured by across this 2 cm length. These voltages are directly compared to the
experimental voltages for benchmarking and comparison purposes. Attached is the source code for this numerical method written in Fortran 77.
Fortran 77 source code

c234567890123456789

program normal

C---------------------------------------------------------------------
C
C This program calculates the current and temperature profile in
C a superconducting sample with a given layer of silver applied
C above the YBCO. Essentially this program is in two parts.
C First, taking the initial temperature conditions, the current
profile is calculated assuming a non-uniform profile of critical
C currents. These values are found from differential equations for
C the current density in the normal metal derived from Gurevich's
C theory and solved with the thomas algorithm. The profile is then
C fed into a series of equations for the temperature of the normal
C metal and superconductor along the samples. A time dependent
C waveform for the current is used to see the time dependence of
C the temperatures under different current pulse magnitudes and
C durations.
C
C list of variables
C -----------------
C i = loop variables
C nx = dimensions of the grid in the x-dir and y-dir
C nbl = number of simultaneous equation & dimension of
blocks
C m, n, istart = dummy variables for the block tri-diag solver
C a, b, c = 2-d blocks for tridiagonal terms of equations
C rts = 1-d array of right hand to solve for deltax

C---------------------------------------------------------------------------------------
---------
C declaration of variables

 implicit none

 integer i, nx, nzones
 integer nwidth, n1, n2, n3, n4, n5, n6, n7, n8, n9
 integer loop, lprint, navg
 parameter (nx = 201, nzones = 10)

 real*8 am(nx), bm(nx), cm(nx)
 real*8 rts(nx), work(nx), x(nx)
 real*8 T(nx), C(nx), In(nx), trak(nx)
 real*8 Ico(nx), Ic(nx), Q(nx), k(nx), res(nx)
 real*8 currnt, i_1, i_2, i_3, i_4
 real*8 k_ybco, k_ag, res_ag, k_kap
 real*8 rho_yb, rho_ag, Cp_yb, Cp_ag, tcon
 real*8 d_ybco, d_ni, d_ag, width, lngth, dx, dx2
 real*8 R_t_i, R_e_i, d_kap, Tbase
real*8 time, dt, tstop, t1, t2, t3, tpulse
real*8 Tc, Top, Tavg, as_bar, an_bar
real*8 rterm1, rterm2, rterm3, rterm4, rterm5
real*8 term1, term2, term3, term4
real*8 Tlead, I_ag, tsum, Im
real*8 V1, V2, V3, V4, V5, V6, V7, V8
real*8 tlim1, tlim2
real*8 Q_cryo, Q_lead
real*8 Tcryo, A_lead, A_l_in, L_lead, m_lead
real*8 V_lead, rho_cu, res_cu, Cp_cu, k_cu
real*8 rho_ni, k_ni, Cp_ni, res_ni
real*8 d_m, d_tot, A_tot, A_m
real*8 alpha, beta, gamma
real*8 Cp_tot, k_tot, res_m, A_ag, A_ni
real*8 d_wire, A_wire, L_wire, L_o, pi, divide
real*8 Ic1, Ic2, Ic3, Ic4, Ic5, Ic6, Ic7, Ic8
real*8 Tavg1, Tavg2, Tavg3, Tavg4, Tavg5, Tavg6, Tavg7, Tavg8
real*8 Iag1, Iag2, Iag3, Iag4, Iag5, Iag6, Iag7, Iag8
real*8 Im1, Im2, Im3, Im4, Im5, Im6, Im7, Im8
real*8 x1, x2, x3, x4, x5, x6, x7, x8
parameter (divide = 0.01)
parameter (pi = 3.14157)
parameter (lngth = 0.05, width = 0.01)
parameter (tstop = 25.0, dt = 0.00005)
parameter (t1 = 0.2, t2 = 0.7, t3 = 25.0)
parameter (i_1 = 2.0, i_2 = 41.5, i_3 = 20.5, i_4 = 0.0)
parameter (Tc = 90.0, Top = 50.0, Tbase = 50.0, Tcryo = 50.0)
parameter (L_o = 2.45e-8)
parameter (tlim1 = 100.0, tlim2 = 83.0)

OPEN (UNIT=1,FILE='nz1.dat',FORM='FORMATTED',STATUS='NEW')
OPEN (UNIT=2,FILE='nz2.dat',FORM='FORMATTED',STATUS='NEW')
OPEN (UNIT=3,FILE='nz3.dat',FORM='FORMATTED',STATUS='NEW')
OPEN (UNIT=4,FILE='nz4.dat',FORM='FORMATTED',STATUS='NEW')

C Initialization of physical parameters of materials
C This is a failsafe if the functions miss a value

k_ybco = 0.46
k_ag = 497.0
k_ni = 400.0
k_kap = 0.5
res_ag = 2.70e-9
rho_yb = 2720.0
rho_ag = 10500.0
rho_cu = 8800.0
rho_ni = 8750.0
Cp_yb = 157.2
Cp_ag = 187.0
Cp_cu = 252.0
Cp_ni = 500.0
d_ag = 8.0e-6
d_ni = 1.35e-5
\[ d_{\text{ybco}} = 5.0 \times 10^{-7} \]
\[ d_{\text{tot}} = d_{\text{ni}} + d_{\text{ag}} + d_{\text{ybco}} \]
\[ A_{\text{ni}} = d_{\text{ni}} \times \text{width} \]
\[ A_{\text{ag}} = d_{\text{ag}} \times \text{width} \]
\[ A_{\text{m}} = (d_{\text{ag}} + d_{\text{ni}}) \times \text{width} \]
\[ A_{\text{tot}} = d_{\text{tot}} \times \text{width} \]
\[ \alpha = d_{\text{ni}} / d_{\text{tot}} \]
\[ \beta = d_{\text{ag}} / d_{\text{tot}} \]
\[ \gamma = d_{\text{ybco}} / d_{\text{tot}} \]
\[ dx = \text{length} / (\text{nx} - 1) \]
\[ dx^2 = dx^2 \]
\[ R_t_i = 5.0 \times 10^{-5} \]
\[ R_e_i = 1.0 \times 10^{-12} \]
\[ \text{time} = 0.0 \]

\[ Q_{\text{lead}} = 0.0 \]
\[ T_{\text{lead}} = 50.0 \]
\[ d_{\text{wire}} = 0.125 / 39.370 \]
\[ L_{\text{wire}} = 1.5 \pi 9.5 / 39.370 \]
\[ A_{\text{wire}} = \pi d_{\text{wire}}^2 / 4.0 \]
\[ A_{\text{l}\_\text{in}} = 0.2 \times 5.0 / 16.0 \]
\[ A_{\text{lead}} = A_{\text{l}\_\text{in}} / 1550.0 \]
\[ L_{\text{lead}} = 1.75 \times 39.370 \]
\[ V_{\text{lead}} = A_{\text{lead}} L_{\text{lead}} \]
\[ m_{\text{lead}} = \rho_{\text{cu}} V_{\text{lead}} \]
\[ t_{\text{pulse}} = 1.0 \]
\[ l_{\text{print}} = 1000 \]
\[ \text{loop} = l_{\text{print}} \]

C check for stability of the selection of the time and position steps
\[ an_{\text{bar}} = (k_{\text{ag}} dt) / (\rho_{\text{ag}} C_{\text{p}}_{\text{ag}} dx^2) \]
\[ \text{print} \ '*, an_{\text{bar}} \]
\[ as_{\text{bar}} = (k_{\text{ybco}} dt) / (\rho_{\text{yb}} C_{\text{p}}_{\text{yb}} dx^2) \]
\[ \text{print} \ '*, as_{\text{bar}} \]

C Initialization of temperatures in sample
\[ \text{do} \ i = 1, \text{nx} \]
\[ T(i) = 50.0 \]
\[ Q(i) = 0.0 \]
\[ \text{end do} \]

C Initialize zone regions
\[ n_{\text{width}} = (\text{nx} - 1) / n_{\text{zones}} \]
\[ n1 = n_{\text{width}} + 1 \]
\[ n2 = n1 + n_{\text{width}} \]
\[ n3 = n2 + n_{\text{width}} \]
\[ n4 = n3 + n_{\text{width}} \]
\[ n5 = n4 + n_{\text{width}} \]
\[ n6 = n5 + n_{\text{width}} \]
\[ n7 = n6 + n_{\text{width}} \]
\[ n8 = n7 + n_{\text{width}} \]
\[ n9 = n8 + n_{\text{width}} \]
x(1) = 0.0
do i = 2, nx
   x(i) = x(i-1)+dx
end do

do i = 1, n1
   Ico(i) = 3.0
end do

do i = n1+1, n2
   Ico(i) = 4.62
end do

do i = n2+1, n3
   Ico(i) = 8.27
end do

do i = n3+1, n4
   Ico(i) = 6.24
end do

do i = n4+1, n5
   Ico(i) = 4.47
end do

do i = n5+1, n6
   Ico(i) = 5.82
end do

do i = n6+1, n7
   Ico(i) = 3.78
end do

do i = n7+1, n8
   Ico(i) = 3.38
end do

do i = n8+1, n9
   Ico(i) = 2.733
end do

do i = n9+1, nx
   Ico(i) = 3.0
end do

C Time iteration loop

do while (time .lt. tstop)
   c if (time.lt.tpulse) then
   c      lprint = 10000
   c   else
   c      lprint = 100000
   c   end if

do i = 1, n1
  if (T(i) .gt. Tc) then
    Ic(i) = -0.5
  else
    Ic(i) = Ico(i)*(Tc-T(i))/(Tc-Top)
  end if
end do

do i = n1+1, n2
  if (T(i) .gt. Tc) then
    Ic(i) = -0.5
  else
    Ic(i) = Ico(i)*(Tc-T(i))/(Tc-Top)
  end if
end do

do i = n2+1, n3
  if (T(i) .gt. Tc) then
    Ic(i) = -0.5
  else
    Ic(i) = Ico(i)*(Tc-T(i))/(Tc-Top)
  end if
end do

do i = n3+1, n4
  if (T(i) .gt. Tc) then
    Ic(i) = -0.5
  else
    Ic(i) = Ico(i)*(Tc-T(i))/(Tc-Top)
  end if
end do

do i = n4+1, n5
  if (T(i) .gt. Tc) then
    Ic(i) = -0.5
  else
    Ic(i) = Ico(i)*(Tc-T(i))/(Tc-Top)
  end if
end do

do i = n5+1, n6
  if (T(i) .gt. Tc) then
    Ic(i) = -0.5
  else
    Ic(i) = Ico(i)*(Tc-T(i))/(Tc-Top)
  end if
end do

do i = n6+1, n7
  if (T(i) .gt. Tc) then
    Ic(i) = -0.5
  else
    Ic(i) = Ico(i)*(Tc-T(i))/(Tc-Top)
  end if
end if
end do

do i = n7+1, n8
  if (T(i) .gt. Tc) then
    Ic(i) = -0.5
  else
    Ic(i) = Ico(i)*(Tc-T(i))/(Tc-Top)
  end if
end do

do i = n8+1, n9
  if (T(i) .gt. Tc) then
    Ic(i) = -0.5
  else
    Ic(i) = Ico(i)*(Tc-T(i))/(Tc-Top)
  end if
end do

do i = n9+1, nx
  if (T(i) .gt. Tc) then
    Ic(i) = -0.5
  else
    Ic(i) = Ico(i)*(Tc-T(i))/(Tc-Top)
  end if
end do

C Setup current conditions

if (time .lt. t1) then
  currnt = i_1
else
  if (time .lt. t2) then
    currnt = i_2
  else
    if (time .lt. t3) then
      currnt = i_3
    else
      currnt = i_4
    end if
  end if
end if

C Calculation of boundary conditions of normal metal that are affected by the current leads attached to the 1st stage of the cryocooler

term1 = 0.6042312*Tlead-0.4852458
term2 = 0.0000026*Tlead**3-0.00191202*Tlead**2
k_cu = term1+term2
res_cu = L_o*Tlead/k_cu
term1 = res_cu*currnt**2*L_lead/A_lead
term2 = k_cu*A_wire/L_wire*(Tlead-Tcryo)
Tlead = Tlead+dt/(m_lead*Cp_cu)*(term1-term2)
T(1) = 55.0
\[
\begin{align*}
T(\text{nx}) &= 55.0 \\
\text{trak}(1) &= 0.0 \\
\text{trak}(\text{nx}) &= 0.0 \\
am(1) &= 0.0 \\
bm(1) &= 1.0 \\
cm(1) &= 0.0 \\
\text{rts}(1) &= T(1) \\
am(\text{nx}) &= 0.0 \\
bm(\text{nx}) &= 1.0 \\
cm(\text{nx}) &= 0.0 \\
\text{rts}(\text{nx}) &= T(\text{nx}) \\
k(1) &= 0.0 \\
k(\text{nx}) &= 0.0 \\
\text{res}(\text{nx}) &= 0.0 \\
\text{res}(1) &= 0.0
\end{align*}
\]

c Setup for matrix of normal metal temperature distribution  
c in central regions  
do i = 2, nx-1

C Setup of thermal and electrical properties of matrix  
C conductor

if (T(i).gt.tlim1) then  
    term1 = 6.667e-7*T(i)**3+2.286e-4*T(i)**2  
    term2 = 1.167e-4*T(i)**3-3.978e-2*T(i)**2  
    k_ag = term1+term2
else  
    term1 = 1.167e-4*T(i)**4-3.978e-2*T(i)**3  
    term2 = 5.088*T(i)**2-290.9*T(i)+6766.0  
    k_ag = term1+term2
end if
if (T(i).gt.tlim2) then  
    k_ybco = 5.272
else  
    term1 = 1.061e-5*T(i)**3-3.184e-3*T(i)**2  
    term2 = 0.2448*T(i)+0.8040  
    k_ybco = term1+term2
end if

\[
\begin{align*}
\text{term1} &= -7.2629e-9*T(i)**5+7.4071e-6*T(i)**4 \\
\text{term2} &= -2.9575e-3*T(i)**3+66.693*T(i)+2398.1 \\
k_{\text{ni}} &= \text{term1}+\text{term2}+\text{term3} \\
k_{\text{tot}} &= 1.00*(\alpha*k_{\text{ni}}+\beta*k_{\text{ag}}+\gamma*k_{\text{ybc}}) \\
k(i) &= k_{\text{tot}} \\
\text{term1} &= -1.044e-7*T(i)**4+9.526e-5*T(i)**3 \\
\text{term2} &= 4.466*T(i)-45.22-2.945e-2*T(i)**2 \\
\text{Cp}_{ag} &= \rho_{ag}*(\text{term1}+\text{term2}) \\
\text{if} (T(i).gt.tlim1) \text{ then}  
    \text{term1} &= 1.275e-5*T(i)**3-1.170e-2*T(i)**2 \\
    \text{term2} &= 4.132*T(i)-102.7 \\
    \text{Cp}_{yb} &= \rho_{yb}*(\text{term1}+\text{term2})
\end{align*}
\]
else
    term1 = 3.063e-6*T(i)**3-4.690e-2*T(i)**2
    term2 = 10.43*T(i)-372.0
    Cp_yb = rho_yb*(term1+term2)
end if

else
    term1 = 2.9702e-5*T(i)**3-0.0228*T(i)**2
    term2 = 6.1354*T(i)-190.0
    Cp_ni = rho_ni*(term1+term2)
    Cp_tot = 1.00*(alpha*Cp_ni+beta*Cp_ag+gamma*Cp_yb)

res_ag = (6.1198e-3*T(i)-0.19267)/100000000.0
term1 = 4.6696e-5*T(i)**2+1.2591e-2*T(i)
res_ni = (term1-0.67255)/100000000.0
term2 = A_ag/res_ag+A_ni/res_ni
res_m = 1/term2
res(i) = res_m

C Setup of tridiagonal terms

am(i) = -k_tot/(2*dx2)
cm(i) = am(i)
term1 = k_tot/dx2+Cp_tot/dt
term2 = res_m*currnt*Ico(i)/(2*A_tot*(Tc-Top))
if (currnt.lt.Ic(i)) then
    bm(i) = term1
    trak(i) = 0.0
else
    if (T(i) .gt. Tc) then
        bm(i) = term1
        trak(i) = 2.0
    else
        bm(i) = term1-term2
        trak(i) = 1.0
    end if
end if
rterm1 = Cp_tot/dt*T(i)
rterm2 = k_tot/(2*dx2)*(T(i+1)-2*T(i)+T(i-1))
if (currnt.lt.Ic(i)) then
    rterm3 = 0.0
    trak(i) = 0.0
else
    if (T(i) .gt. Tc) then
        rterm3 = res_m*currnt**2/(A_tot)
        trak(i) = 2.0
    else
        term1 = res_m*currnt**2/(A_tot)
term2 = A_tot*(Tc-Top)
term3 = res_m*currnt*Ico(i)*Tc/term2
term4 = res_m*currnt*Ico(i)*T(i)/(2*term2)
        trak(i) = 1.0
        rterm3 = term1-term3+term4
    end if
end if
C(i) = rterm1
\[
k(i) = r_{\text{term2}} \\
Q(i) = r_{\text{term3}} \\
rt(i) = r_{\text{term1}} + r_{\text{term2}} + r_{\text{term3}}
\]
end do

call thomas\(am, bm, cm, work, rt\), \(nx\)

do \(i = 1, nx\)
\(T(i) = rt(i)\)
end do

C Voltage drop calculation
\[
tsum = 0.0 \\
do \(i = n1+1, n2\)
\(tsum = tsum + T(i)\)
end do
Tavg1 = tsum/(nwidth) \\
ng = (n1+n2)/2 \\
x1 = x(navg) \\
Ic1 = Ico(navg)*(Tc-Tavg1)/(Tc-Top) \\
if (currnt.lt.Ic1) then \\
Im = 0.0 \\
else \\
if (Tavg1.lt.Tc) then \\
Im = currnt-Ic1 \\
else \\
Im = currnt \\
end if
end if
\[
res_{ag} = (6.1198e-3*Tavg1-0.19267)/100000000.0 \\
term1 = 4.6696e-5*Tavg1**2+1.2591e-2*Tavg1 \\
res_{ni} = (term1-0.67255)/100000000.0 \\
I_{ag} = Im*res_{ni}/A_{ni}/(res_{ni}/A_{ni}+res_{ag}/A_{ag}) \\
Im1 = Im \\
Iag1 = I_{ag} \\
V1 = res_{ag}*I_{ag}*(x(n2)-x(n1))/A_{ag}
\]
\[
tsum = 0.0 \\
do \(i = n2+1, n3\)
\(tsum = tsum + T(i)\)
end do
Tavg2 = tsum/(nwidth) \\
ng = (n2+n3)/2 \\
x2 = x(navg) \\
Ic2 = Ico(navg)*(Tc-Tavg2)/(Tc-Top) \\
if (currnt.lt.Ic2) then \\
Im = 0.0 \\
else \\
if (Tavg2.lt.Tc) then \\
Im = currnt-Ic2 \\
else \\
Im = currnt \\
end if 
end if


\[
\text{res\_ag} = \frac{(6.1198e-3*Tavg2-0.19267)}{100000000.0}
\]

\[
\text{term1} = 4.6696e-5*Tavg2^{2}+1.2591e-2*Tavg2
\]

\[
\text{res\_ni} = \frac{(term1-0.67255)}{100000000.0}
\]

\[
\text{I\_ag} = \frac{\text{Im}\times\text{res\_ni}/\text{A\_ni}/(\text{res\_ni/\text{A\_ni}+res\_ag/\text{A\_ag}})}{100000000.0}
\]

\[
\text{I\_ag2} = \text{I\_ag}
\]

\[
\text{V2} = \text{res\_ag*I\_ag*(x(n3)-x(n2))/A\_ag}
\]

\[
\text{tsum} = 0.0
\]

do \ i = n3+1, n4

\[
\text{tsum} = \text{tsum+T(i)}
\]
end do

\[
\text{Tavg3} = \text{tsum/(nwidth)}
\]

\[
\text{navg} = (n3+n4)/2
\]

\[
\text{x3} = \text{x(navg)}
\]

\[
\text{Ic3} = \text{Ico(navg)}*(\text{Tc-Tavg3})/(\text{Tc-Top})
\]

if (\text{currnt.lt.Ic3}) then

\[
\text{Im} = 0.0
\]
else

if (\text{Tavg3.lt.Tc}) then

\[
\text{Im} = \text{currnt-Ic3}
\]
else

\[
\text{Im} = \text{currnt}
\]
end if
end if

\[
\text{res\_ag} = \frac{(6.1198e-3*Tavg3-0.19267)}{100000000.0}
\]

\[
\text{term1} = 4.6696e-5*Tavg3^{2}+1.2591e-2*Tavg3
\]

\[
\text{res\_ni} = \frac{(term1-0.67255)}{100000000.0}
\]

\[
\text{I\_ag} = \frac{\text{Im}\times\text{res\_ni}/\text{A\_ni}/(\text{res\_ni/\text{A\_ni}+res\_ag/\text{A\_ag}})}{100000000.0}
\]

\[
\text{I\_ag3} = \text{I\_ag}
\]

\[
\text{V3} = \text{res\_ag*I\_ag*(x(n4)-x(n3))/A\_ag}
\]

\[
\text{tsum} = 0.0
\]

do \ i = n4+1, n5

\[
\text{tsum} = \text{tsum+T(i)}
\]
end do

\[
\text{Tavg4} = \text{tsum/(nwidth)}
\]

\[
\text{navg} = (n4+n5)/2
\]

\[
\text{x4} = \text{x(navg)}
\]

\[
\text{Ic4} = \text{Ico(navg)}*(\text{Tc-Tavg4})/(\text{Tc-Top})
\]

if (\text{currnt.lt.Ic4}) then

\[
\text{Im} = 0.0
\]
else

if (\text{Tavg4.lt.Tc}) then

\[
\text{Im} = \text{currnt-Ic4}
\]
else

\[
\text{Im} = \text{currnt}
\]
end if
end if

\[
\text{res\_ag} = \frac{(6.1198e-3*Tavg4-0.19267)}{100000000.0}
\]

\[
\text{term1} = 4.6696e-5*Tavg4^{2}+1.2591e-2*Tavg4
\]

\[
\text{res\_ni} = \frac{(term1-0.67255)}{100000000.0}
\]

\[
\text{I\_ag} = \frac{\text{Im}\times\text{res\_ni}/\text{A\_ni}/(\text{res\_ni/\text{A\_ni}+res\_ag/\text{A\_ag}})}{100000000.0}
\]
\[ I_{m4} = I_m \]
\[ I_{ag4} = I_{ag} \]
\[ V_4 = \text{res}_a g I_{ag} (x(n5) - x(n4))/A_{ag} \]

\[ \text{tsum} = 0.0 \]
\[ \text{do } i = n5+1, n6 \]
\[ \quad \text{tsum} = \text{tsum} + T(i) \]
\[ \text{end do} \]
\[ T_{avg5} = \text{tsum}/(\text{nwidth}) \]
\[ \text{navg} = (n5+n6)/2 \]
\[ x_5 = x(\text{navg}) \]
\[ Ic_5 = Ico(\text{navg})*(Tc-T_{avg5})/(Tc-Top) \]
\[ \text{if } (\text{currnt} \lt Ic_5) \text{ then} \]
\[ \quad \text{Im} = 0.0 \]
\[ \text{else} \]
\[ \quad \text{if } (T_{avg5} \lt Tc) \text{ then} \]
\[ \quad \quad \text{Im} = \text{currnt} - Ic_5 \]
\[ \quad \text{else} \]
\[ \quad \quad \text{Im} = \text{currnt} \]
\[ \text{end if} \]
\[ \text{end if} \]
\[ \text{res}_a g = (6.1198e-3*T_{avg5} - 0.19267)/100000000.0 \]
\[ \text{term1} = 4.6696e-5*T_{avg5}^2 + 1.2591e-2*T_{avg5} \]
\[ \text{res}_n i = (\text{term1} - 0.67255)/100000000.0 \]
\[ I_{ag} = \text{Im} * \text{res}_n i / A_{ni} / (\text{res}_n i / A_{ni} + \text{res}_a g / A_{ag}) \]
\[ I_m5 = \text{Im} \]
\[ I_{ag5} = I_{ag} \]
\[ V_5 = \text{res}_a g I_{ag} (x(n6) - x(n5))/A_{ag} \]

\[ \text{tsum} = 0.0 \]
\[ \text{do } i = n6+1, n7 \]
\[ \quad \text{tsum} = \text{tsum} + T(i) \]
\[ \text{end do} \]
\[ T_{avg6} = \text{tsum}/(\text{nwidth}) \]
\[ \text{navg} = (n6+n7)/2 \]
\[ x_6 = x(\text{navg}) \]
\[ Ic_6 = Ico(\text{navg})*(Tc-T_{avg6})/(Tc-Top) \]
\[ \text{if } (\text{currnt} \lt Ic_6) \text{ then} \]
\[ \quad \text{Im} = 0.0 \]
\[ \text{else} \]
\[ \quad \text{if } (T_{avg6} \lt Tc) \text{ then} \]
\[ \quad \quad \text{Im} = \text{currnt} - Ic_6 \]
\[ \quad \text{else} \]
\[ \quad \quad \text{Im} = \text{currnt} \]
\[ \text{end if} \]
\[ \text{end if} \]
\[ \text{res}_a g = (6.1198e-3*T_{avg6} - 0.19267)/100000000.0 \]
\[ \text{term1} = 4.6696e-5*T_{avg6}^2 + 1.2591e-2*T_{avg6} \]
\[ \text{res}_n i = (\text{term1} - 0.67255)/100000000.0 \]
\[ I_{ag} = \text{Im} * \text{res}_n i / A_{ni} / (\text{res}_n i / A_{ni} + \text{res}_a g / A_{ag}) \]
\[ I_m6 = \text{Im} \]
\[ I_{ag6} = I_{ag} \]
\[ V_6 = \text{res}_a g I_{ag} (x(n7) - x(n6))/A_{ag} \]
tsum = 0.0
do i = n7+1, n8
   tsum = tsum+T(i)
end do
Tavg7 = tsum/(nwidth)
navg = (n7+n8)/2
x7 = x(navg)
Ic7 = Ico(nav g)*(Tc-Tavg7)/(Tc-Top)
if (currnt.lt.Ic7) then
   Im = 0.0
else
   if (Tavg7.lt.Tc) then
      Im = currnt-Ic7
   else
      Im = currnt
   end if
end if
res_ag = (6.1198e-3*Tavg7-0.19267)/100000000.0
term1 = 4.6696e-5*Tavg7**2+1.2591e-2*Tavg7
res_ni = (term1-0.67255)/100000000.0
I_ag = Im*res_ni/A_ni/(res_ni/A_ni+res_ag/A_ag)
Im7 = Im
Iag7 = I_ag
V7 = res_ag*I_ag*(x(n8)-x(n7))/A_ag

if (loop.eq.lprint) then
   print *, time
   write (2, 350) time, x1, V1, Tavg1
write (2, 350) time, x2, V2, Tavg2
write (2, 350) time, x3, V3, Tavg3
write (2, 350) time, x4, V4, Tavg4
write (2, 350) time, x5, V5, Tavg5
write (2, 350) time, x6, V6, Tavg6
write (2, 350) time, x7, V7, Tavg7
write (2, 350) time, x8, V8, Tavg8
do i = 1, nx
  write (1, 250) time, x(i), current, Q(i), Ic(i), T(i)
  write (3, 450) time, x(i), C(i), trak(i), Q(i)
  write (4, 450) time, x(i), am(i), bm(i), cm(i)
end do
else
  loop = loop + 1
end if
time = time + dt
end do

250 format (f9.6,3x,f9.6,3x,f9.6,3x,e15.6,3x,f15.6,3x,f15.6)
350 format (f9.6,2x,f8.5,2x,e15.6,2x,f15.6)
450 format (f9.6,2x,f8.5,2x,e15.6,2x,e15.6,2x,e15.6)
endfile (unit=1)
close (unit=1)
endfile (unit=2)
close (unit=2)
endfile (unit=3)
close (unit=3)
endfile (unit=4)
close (unit=4)
end

---------------------------------------------------------------
subroutine thomas (a,b,c,cwrk,r,n)
---------------------------------------------------------------
implicit none
integer i,n
real*8 a(n),b(n),c(n),cwrk(n),r(n),fac
---------------------------------------------------------------
c forward elimination
fac = 1.d0/b(1)
cwrk(1) = c(1) * fac
r(1) = r(1) * fac
do i = 2,n
    fac = 1.d0/( b(i) - a(i)*cwrk(i-1) )
    cwrk(i) = c(i) * fac
    r(i) = ( r(i) - r(i-1) * a(i) ) * fac
end do

c-----------------------------------------------------------------------
c back substitution
c put solution into r array, r(n) already has solution
do i = n-1,1,-1
    r(i) = r(i) - cwrk(i)*r(i+1)
end do

c-----------------------------------------------------------------------
return
end

c=======================================================================